

Этапы решения задач о гравитационном поле

1. Электрогравитика в кеплерской метрике, т.е. в пространстве с (заданной, пока) гравитацией.
2. Гравитация, порождаемая электромагнитным полем, т.е. подстановка тензора энергии-импульса электромагнитного поля в уравнение Эйнштейна и нахождение его решения, т.е. канонической кривизны $R(x^M)$, $M=0,1,2,3$.
3. Подстановка 2. в 1., т.е. кеплерскую кривизну, зависящую от времени, для электромагнитной волны $H(t) e^{i\omega t}$, подставившем в левую часть н.л., при этом эта кривизна выражена в переменных \vec{E} и \vec{H} . Из этих более полных уравнений Максвелла находим волновое уравнение и ищем его решение в виде $f(t) e^{i\omega t}$, где $f(t)$ — неизвестная функция. Рецепт поиска решения таков: подставив $f(t) e^{i\omega t}$ группировать члены так, чтобы выделить каноническое уравнение Гельмгольца (если это возможно) и эту часть уравнения решить, т.к. в правой части уравнения Гельмгольца стоит нуль. Оставшееся уравнение решаем и находим $f(t)$ и $\omega(t)$.

Примечание к пп. 2, 3: частота — функция времени.

4. Демонстрация утверждения:

любой поток энергии в результате самогенерации через гравитационное поле квантуется (квантуется себя).

5. Вычисление фундаментальных констант:

$h, (e, m_{\text{электрон}} \text{ для симметричных полей}),$

используя различные асимптотические решения

более простых уравнений, определить E или $\hbar \omega$.

6. Триггерная линия.

Центранное - симметричное прав. поле.

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- 1. Распределение ρ -ла - центр. - симм.
- 2. Дивиденс ρ -ла - центр. симм.

Возможные "сферически" пространственными координатами. Наиболее общим выражением для метрики является:

$$dS^2 = h(\bar{z}, t) dz^2 + \kappa(\bar{z}, t) (\sin^2 \Theta \cdot d\varphi^2 + d\Theta^2) + \ell(\bar{z}, t) dt^2 + a(\bar{z}, t) dz dt, \tag{1}$$

где a, h, κ, ℓ - некоторые ср-ции.

В книге Чандрасекара доказываеся, что можно преобразовать координаты \bar{z} и t таким образом, что $a(\bar{z}, t) = 0$, при этом $\kappa(\bar{z}, t) = z^2$. Берем h и ℓ заменим в виде $-e^\lambda$ и e^ν .

$$dS^2 = e^\nu e^z dt^2 - z^2 (d\Theta^2 + \sin^2 \Theta d\varphi^2) - e^\lambda dz^2 \tag{2}$$

Отличие от нуль компонент метрического тензора

$$g_{00} = e^\nu, g_{11} = -e^\lambda, g_{22} = -z^2, g_{33} = -z^2 \sin^2 \Theta;$$

$$g^{00} = e^{-\nu}, g^{11} = -e^{-\lambda}, g^{22} = -z^{-2}, g^{33} = -z^{-2} \sin^{-2} \Theta;$$

Тогда выражение в параметрах $dS^2 = g_{ik} dx^i dx^k$ по x^0, x^1, x^2, x^3 соответственно ct, z, Θ, φ .

То формуле $\Gamma_{ke}^i = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^e} + \frac{\partial g_{me}}{\partial x^k} - \frac{\partial g_{ke}}{\partial x^m} \right)$ (2)

числами числом Кривизны:

$$\left. \begin{aligned} \Gamma_{11}^1 &= \frac{\lambda'}{2}; \quad \Gamma_{20}^0 = \frac{\nu'}{2}; \quad \Gamma_{33}^2 = -\sin \theta \cos \theta; \\ \Gamma_{22}^0 &= \frac{\lambda}{2} e^{\lambda-\nu}; \quad \Gamma_{22}^1 = -2e^{-\lambda}; \quad \Gamma_{00}^1 = \frac{\nu'}{2} e^{\nu-\lambda}; \\ \Gamma_{22}^2 &= \Gamma_{13}^3 = \frac{1}{2}; \quad \Gamma_{23}^3 = \cotg \theta; \quad \Gamma_{00}^0 = \frac{\nu}{2}; \\ \Gamma_{10}^1 &= \frac{\lambda}{2}; \quad \Gamma_{33}^1 = -2 \sin^2 \theta \cdot e^{-\lambda}. \end{aligned} \right\} \begin{array}{l} 15 \text{ компонент} \\ (3) \end{array}$$

То формуле $R_{ik} = \frac{\partial \Gamma_{ik}^e}{\partial x^e} - \frac{\partial \Gamma_{ie}^k}{\partial x^k} + \Gamma_{ik}^e \Gamma_{em}^m - \Gamma_{ie}^m \Gamma_{km}^e$

числами компонента тензора кривизны и поставившие в уравнение Эйнштейна $R_{ik} = \frac{8\pi k}{c^4} \left(T_{ik} - \frac{1}{2} g_{ik} T \right)$

получаем

$$\frac{8\pi k}{c^4} T_1^1 = -e^{-\lambda} \left(\frac{\nu'}{2} + \frac{1}{2^2} \right) + \frac{1}{2^2}, \quad (4)$$

$$\begin{aligned} \frac{8\pi k}{c^4} T_2^2 = \frac{8\pi k}{c^4} T_3^3 &= -\frac{1}{2} e^{-\lambda} \left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{2} - \frac{\nu' \lambda'}{2} \right) + \\ &+ \frac{1}{2} e^{-\nu} \left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda} \dot{\nu}}{2} \right), \end{aligned} \quad (5)$$

$$\frac{8\pi k}{c^4} T_0^0 = -e^{-\lambda} \left(\frac{1}{2^2} - \frac{\lambda'}{2} \right) + \frac{1}{2^2}, \quad (6)$$

$$\frac{8\pi k}{c^4} T_0^3 = -e^{-\lambda} \frac{\dot{\lambda}}{2} \quad (7)$$

Διασυνιστώσα $T^i_k = 0$:

(3)

$$e^{-\lambda} \left(\frac{\nu'}{2} + \frac{1}{2z^2} \right) - \frac{1}{2z^2} = 0 \quad (8)$$

$$e^{-\lambda} \left(\frac{\lambda'}{2} - \frac{1}{2z^2} \right) + \frac{1}{2z^2} = 0 \quad (9)$$

$$\dot{\lambda} = 0 \quad (10)$$

λ - η σταθερά t .

$$(8) + (9) : \lambda' + \nu' = 0, \text{ i.e.}$$

$$\lambda + \nu = f(t) - \text{const} \quad (11)$$

Από ds^2 στο ds^2 (2) παίρνουμε το εξής αποτέλεσμα:

$$\nu + f(t) - \text{const} = 0, \Rightarrow \lambda + \nu = 0$$

Από (9) :

$$e^{-\lambda} = e^{\nu} = 1 + \frac{\text{const}}{2} \quad (12)$$

Όταν $z \rightarrow \infty$ $e^{-\lambda} = e^{\nu} = 1$, i.e. $\text{const} = 0$.

const εξαρτάται από το z και δίνεται από

$$g_{00} = 1 + \frac{2\psi}{c^2}, \text{ where } \psi = -\frac{km}{z} - \text{gravitational potential}, \Rightarrow$$

$$\text{const} = -\frac{2km}{c^2} = z_g - \text{gravitational radius}. \quad (13)$$

Τελικό αποτέλεσμα:

$$ds^2 = \left(1 - \frac{z_g}{z}\right) e^{2t} dt^2 - z^2 (\sin^2 \theta \cdot d\varphi^2 + d\theta^2) - \frac{dz^2}{1 - \frac{z_g}{z}} \quad (14)$$

Получили ур-е y -сим. прел. масс b b -ге (4)
в компьютерной системе отсчёта.

Запишем метрику в самом общем виде:

$$dS^2 = h dz^2 + \kappa (\sin^2 \theta d\varphi^2 + d\theta^2) + \ell dt^2 + a dz dt,$$

Сон. сист. отсч. называется система с $a=0$ и рагу-
 альной составляющей скорости $= 0$ (от-е конн. и так $= 0$).

Приведем коор-ты z и t гон-му преобр-ю $z = z(z')$,

$t = t(t')$ и подставим вдранные таким образом время

и пространств координаты r и R , а коэфф-ты h, κ, ℓ -
 соответственно: $-e^\lambda, -e^\mu, e^\nu$ (λ, μ, ν - ф-ция R и t).

Тогда:

$$dS^2 = e^2 e^\nu dt'^2 - e^\lambda dr^2 - e^\mu (d\theta^2 + \sin^2 \theta d\varphi^2)$$

(1)

В эту сур-е ССО тензор энергии-импульса имеет
 компоненты:

$$T_0^0 = \varepsilon, \quad T_1^1 = T_2^2 = T_3^3 = -p$$

$$\Gamma_{ke}^i = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^e} + \frac{\partial g_{me}}{\partial x^k} - \frac{\partial g_{ke}}{\partial x^m} \right)$$

$$g_{im} \neq 0$$

$$g_{00} = e^\nu, \quad g_{11} = -e^\lambda, \quad g_{22} = -e^\mu, \quad g_{33} = -e^\mu \sin^2 \theta$$

$$g^{00} = e^{-\nu}, \quad g^{11} = -e^{-\lambda}, \quad g^{22} = -e^{-\mu}, \quad g^{33} = -e^{-\mu} \sin^{-2} \theta$$

Одномерный нестационарный случай

(1)

1. В нашем случае \exists четная зеркальная симметрия, поэтому метрика должна оставаться неизменной при замене $y \rightarrow -y$, $z \rightarrow -z$.

В силу этого

$$g_{23} = 0$$

$$g_{02} = g_{03} = 0$$

$$g_{12} = g_{13} = 0$$

2. Угловые функции α и ω :

$$ds^2 = \alpha dt^2 + 2\beta dt dx + \gamma dx^2 + \omega(dy^2 + dz^2)$$

$$\text{где } \begin{cases} \alpha = \alpha(t, x) \\ \beta = \beta(t, x) \\ \gamma = \gamma(t, x) \\ \omega = \omega(t, x) \end{cases}, \quad \begin{cases} \alpha = g_{00} \\ \beta = g_{01} \\ \gamma = g_{11} \\ \omega = g_{22} = g_{33} \end{cases}$$

3. Введем символы Кристоффеля. Для этого

найдем g^{em} - обратный к g_{em} .

$$g_{em} = \left[\begin{array}{cc|cc} \alpha & \beta & 0 & 0 \\ \beta & \gamma & 0 & 0 \\ \hline 0 & 0 & \omega & 0 \\ 0 & 0 & 0 & \omega \end{array} \right]$$

$g_{em} g^{mk} = \delta_e^k$ - константы обратности тензоров.

Умножим g^{mk} на обе:

(2)

$$\left[\begin{array}{cc|c} x & y & 0 \\ y & z & 0 \\ \hline 0 & u & 0 \\ 0 & 0 & u \end{array} \right]$$

$$\left(\begin{array}{cc|c} \alpha\beta & 0 & 0 \\ \beta\delta & 0 & 0 \\ \hline 0 & \omega 0 & 0 \\ 0 & 0 & \omega u \end{array} \right) \left(\begin{array}{cc|c} x & y & 0 \\ y & z & 0 \\ \hline 0 & u & 0 \\ 0 & 0 & u \end{array} \right) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ & & 1 \end{pmatrix}$$

Преобразуем матрицу

$$\left(\begin{array}{cc|cc} (\alpha x + \beta y) & (\alpha y + \beta z) & & \\ (\beta x + \delta y) & (\beta y + \delta z) & & \\ \hline & & \omega u & 0 \\ & & 0 & \omega u \end{array} \right) = \begin{pmatrix} 1 & 1 & 0 \\ & 1 & 1 \\ & & 1 \end{pmatrix}$$

Получим систему уравнений:

$$\begin{cases} \alpha x + \beta y = 1 \\ \alpha y + \beta z = 0 \\ \beta x + \delta y = 0 \\ \beta y + \delta z = 1 \\ \omega u = 1 \end{cases}$$

Решим эту систему получим:

$$x = \frac{\delta}{\alpha\delta - \beta^2}$$

$$y = -\frac{\beta}{\alpha\delta - \beta^2}$$

$$z = \frac{\alpha}{\alpha\delta - \beta^2}$$

$$u = \frac{1}{\omega}$$

Таким образом

(3)

$$\left[\begin{array}{cc|cc} \frac{\gamma}{2\gamma - \beta^2} & -\frac{\beta}{2\gamma - \beta^2} & & \\ \frac{\beta}{2\gamma - \beta^2} & \frac{\alpha}{2\gamma - \beta^2} & & \\ \hline & & \frac{1}{\omega} & 0 \\ & & 0 & \frac{1}{\omega} \end{array} \right] = g^{em}.$$

В результате ненулевые символы Кристоффеля имеют вид:

Γ_{00}^0	...
Γ_{01}^0	...
...	...
...	...
	...
	...
	...
...	...
	...
	...
Γ_{13}^3	...

Вниманием тензора тензор Риззи:

$$R_{\alpha\beta} = \frac{\partial \Gamma_{\beta\alpha}^{\sigma}}{\partial x^{\sigma}} - \frac{\partial \Gamma_{\sigma\alpha}^{\beta}}{\partial x^{\sigma}} + \Gamma_{\sigma\gamma}^{\sigma} \Gamma_{\beta\alpha}^{\gamma} - \Gamma_{\beta\gamma}^{\sigma} \Gamma_{\sigma\alpha}^{\gamma}$$

Кривые компоненты, тензор Рунге имеет вид:

(4)

$$R_{00} = \frac{\partial \Gamma_{00}^1}{\partial x} - \frac{\partial \Gamma_{01}^1}{\partial t} - 2 \frac{\Gamma_{02}^2}{\partial t} +$$

$$+ \Gamma_{01}^1 \Gamma_{00}^0 + 2 \Gamma_{02}^2 \Gamma_{00}^0 + \Gamma_{11}^1 \Gamma_{00}^1 + 2 \Gamma_{12}^2 \Gamma_{00}^1 -$$

$$- \Gamma_{01}^0 \Gamma_{00}^1 - \Gamma_{01}^1 \Gamma_{01}^1 - 2 \Gamma_{02}^2 \Gamma_{02}^2.$$

$$R_{01} = \frac{\partial \Gamma_{01}^0}{\partial t} - \frac{\partial \Gamma_{00}^0}{\partial x} - 2 \frac{\partial \Gamma_{02}^2}{\partial x} +$$

$$+ 2 \Gamma_{02}^2 \Gamma_{01}^0 + \Gamma_{01}^0 \Gamma_{01}^1 + 2 \Gamma_{12}^2 \Gamma_{01}^1 - \Gamma_{11}^0 \Gamma_{00}^1 - 2 \Gamma_{12}^2 \Gamma_{02}^2.$$

$$R_{11} = \frac{\partial \Gamma_{11}^0}{\partial t} - \frac{\partial \Gamma_{01}^0}{\partial x} - 2 \frac{\partial \Gamma_{12}^2}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{11}^0 + 2 \Gamma_{02}^2 \Gamma_{11}^0 + \Gamma_{01}^0 \Gamma_{11}^1 + 2 \Gamma_{12}^2 \Gamma_{11}^1 -$$

$$- \Gamma_{01}^0 \Gamma_{01}^0 - \Gamma_{11}^0 \Gamma_{01}^1 - 2 \Gamma_{12}^2 \Gamma_{12}^2.$$

$$R_{22} = \frac{\partial \Gamma_{22}^0}{\partial t} + \frac{\partial \Gamma_{22}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{22}^0 + \Gamma_{01}^1 \Gamma_{22}^0 + \cancel{\Gamma_{03}^3 \Gamma_{22}^0} + \Gamma_{01}^0 \Gamma_{22}^1 +$$

$$+ \Gamma_{11}^1 \Gamma_{22}^1 + \cancel{\Gamma_{13}^3 \Gamma_{22}^1} - \cancel{\Gamma_{22}^0 \Gamma_{02}^2} - \cancel{\Gamma_{22}^1 \Gamma_{12}^2}.$$

$$R_{22} = R_{33}$$

$$R_{33} = \frac{\partial \Gamma_{33}^0}{\partial t} + \frac{\partial \Gamma_{33}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{33}^0 + \Gamma_{01}^1 \Gamma_{33}^0 + \cancel{\Gamma_{02}^2 \Gamma_{33}^0} + \Gamma_{01}^0 \Gamma_{33}^1 +$$

$$+ \Gamma_{11}^1 \Gamma_{33}^1 + \cancel{\Gamma_{12}^2 \Gamma_{33}^1} - \cancel{\Gamma_{13}^3 \Gamma_{33}^1} - \cancel{\Gamma_{32}^0 \Gamma_{03}^3}.$$

Рассмотрим более простую задачу, а именно:

метрический тензор выберем в виде

$$g_{\alpha\beta} = \begin{pmatrix} 2 & 0 \\ 0 & \delta \end{pmatrix}$$

Это значит, что мы рассматриваем не одномерный случай в 4-мерном ир.-вр., а одномерный случай в 2-мерном ир.-вр., когда \exists только одна ир.-в координата. В результате система уравнений Ланжвэна должна превратиться в одно уравнение.

Обратный тензор $g^{\alpha\beta} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{\delta} \end{pmatrix}$.

Символы Кристоффеля:

$$\begin{pmatrix} 1 \\ 11 \end{pmatrix} = \frac{1}{2} \cdot \frac{d}{dt}$$

$$\begin{pmatrix} 1 \\ 12 \end{pmatrix} = \frac{1}{2} \cdot \frac{d'}{dt}$$

$$\begin{pmatrix} 1 \\ 22 \end{pmatrix} = -\frac{1}{2} \cdot \frac{\ddot{\delta}}{\delta}$$

$$\begin{pmatrix} 2 \\ 11 \end{pmatrix} = -\frac{1}{2} \cdot \frac{d'}{\delta}$$

$$\begin{pmatrix} 2 \\ 22 \end{pmatrix} = \frac{1}{2} \cdot \frac{\dot{\delta}}{\delta}$$

$$\begin{pmatrix} 2 \\ 22 \end{pmatrix} = \frac{1}{2} \cdot \frac{\delta'}{\delta}$$

Тензор Римана

$$R_{2212}^1 = \frac{\partial(\frac{1}{22})}{\partial x^1} - \frac{\partial(\frac{1}{12})}{\partial x^2} +$$

$$+ \begin{pmatrix} 1 \\ 11 \end{pmatrix} \begin{pmatrix} 1 \\ 22 \end{pmatrix} + \begin{pmatrix} 1 \\ 12 \end{pmatrix} \begin{pmatrix} 2 \\ 22 \end{pmatrix} - \begin{pmatrix} 1 \\ 22 \end{pmatrix} \begin{pmatrix} 1 \\ 12 \end{pmatrix} - \begin{pmatrix} 1 \\ 22 \end{pmatrix} \begin{pmatrix} 2 \\ 11 \end{pmatrix}$$

$$R_{2212}^1 = -\frac{1}{2} \frac{\ddot{\delta}}{\delta} - \frac{1}{2} \frac{d''}{\delta} + \frac{1}{4} \frac{\dot{\delta} \dot{\delta}}{\delta} + \frac{1}{4} \frac{d'^2}{\delta} +$$

$$+ \frac{1}{4} \frac{\dot{\delta}^2}{\delta} + \frac{1}{4} \frac{d' \delta'}{\delta}$$

Тензор Риччи

$$R_{22} = \frac{1}{\delta} R_{2212} \quad ; \quad R_{22} = \frac{1}{2} R_{2212} \quad ; \quad R_{12} = 0$$

$$R = \frac{2}{\delta} R_{2212} \quad - \text{ скалярная кривизна.}$$

Возьмем направление электромагнитного поля

(1)

в направлении:

$$L = \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + R/2\alpha,$$

где $F_{\mu\nu}$ - тензор электромагнитного поля, R - скалярная кривизна отвечающая соответствующему гравитационному полю электромагнитного поля, α - константа, зависящая от выбора системы единиц (для системы СГС $\alpha = 4\pi/c^4$, γ - гравитационная постоянная, c - скорость света в вакууме).

Скалярная кривизна связана с тензором энергии-импульса вращением [д.д. т. 2, 1988, с. 357]:

$$R = -\alpha T = 0$$

где $T = T^i_i$ - тензор энергии-импульса электромагнитного поля, но $T^i_i = 0$ [д.д. т. 2, 1988, с. 114], поэтому

выберем

$$T^{ik} = \frac{1}{4\pi} \left(-F^{ie} F^k_e + \frac{1}{4} g^{ik} F_{em} F^{em} \right)$$

где g^{ik} соответствует соответствующему гравитационному полю электромагнитного поля.

Надо решить две проблемы:

1. Взять из T^{ik} скаляр и добавить его на скалярную кривизну.
2. Взять g^{ik} .

$$L = \cancel{L_{\text{d. n.}}} + L_{\text{d. n.}}$$



$$T_{ik} \neq 0$$

Y

$$T_{ik} = \frac{1}{2} (-T_{ik} T_{ik} + \dots)$$

$$T_{ik} = 0$$

$$C = -T_{ik} = 0$$

$$T_{ik} = \dots$$

$$T^{ik} = \frac{1}{4\pi} \left(-F^{il} F^{lk} + \frac{1}{4} g^{ik} F_{em} F^{em} \right), \quad \underline{g^{ik} = ?} \quad (2)$$

$R_{ik} = \frac{8\pi G}{c^4} T_{ik}$ — уравнение Эйнштейна при $R=0$.

$$R_{ik} = \frac{\partial \Gamma^e_{ik}}{\partial x^e} - \frac{\partial \Gamma^e_{ie}}{\partial x^k} + \Gamma^e_{ik} \Gamma^m_{em} - \Gamma^m_{il} \Gamma^e_{km}$$

$$\Gamma^i_{ke} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^e} + \frac{\partial g_{me}}{\partial x^k} - \frac{\partial g_{ke}}{\partial x^m} \right)$$

$$\frac{\partial g^{ik}}{\partial x^e} = -\Gamma^i_{me} g^{mk} - \Gamma^k_{me} g^{im}$$

Из этих уравнений можно вывести g^{ik} .

$g_{\alpha\beta}$
 $F_{\alpha\beta}$
 A_α

Derivative Lagrangian

$$S = \int d^4x \sqrt{-g} \left\{ R - \frac{1}{32\pi} F_{\mu\nu} F_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta} \right\}$$

Динамические уравнения

$$\text{Классич. } G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{1}{4\pi} \left(F_{\alpha\nu} F_{\beta}{}^\nu - \frac{1}{4} g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu} \right)$$

S, \mathcal{L} рассматривать как функции переменных, там и вводить их.

$$e^{\frac{i}{\hbar} S[g_{\alpha\beta}, A_\alpha]}$$

$$G_{\alpha\beta} = 0 \Rightarrow F_{\alpha\nu} = 0$$

$$R^{\lambda}_{\sigma\mu\beta} = \frac{\partial \Gamma^{\lambda}_{\beta\sigma}}{\partial x^{\mu}} - \frac{\partial \Gamma^{\lambda}_{\mu\sigma}}{\partial x^{\beta}} + \Gamma^{\lambda}_{\alpha\mu} \Gamma^{\alpha}_{\beta\sigma} - \Gamma^{\lambda}_{\beta\mu} \Gamma^{\alpha}_{\alpha\sigma} =$$

$$R_{0101}, R_{0102}, R_{0103}, \dots$$

$$R_{0202}, R_{0203}, \dots$$

$$R_{0303}$$

$$R_{1212}, R_{1213}, R_{1313}$$

$$R_{2323}, \dots$$

$$1) \Gamma_{00}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m0}}{\partial x^0} + \frac{\partial g_{m0}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^m} \right) =$$

$$= g^{0m} \dot{g}_{m0} - \frac{1}{2} (\dot{g}_{00} + \dot{g}'_{00}) g^{0m} = \underline{g^{0m} (\dot{g}_{m0} - \frac{1}{2} (\dot{g}_{00} + \dot{g}'_{00}))}$$

$$2) \Gamma_{01}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m0}}{\partial x^1} + \frac{\partial g_{m1}}{\partial x^0} - \frac{\partial g_{01}}{\partial x^m} \right) =$$

$$= \frac{1}{2} g^{0m} \dot{g}'_{m0} + \frac{1}{2} g^{0m} \dot{g}'_{m1} - \frac{1}{2} g^{0m} (\dot{g}_{01} + \dot{g}'_{01}) =$$

$$= \underline{\frac{1}{2} g^{0m} (\dot{g}'_{m0} + \dot{g}'_{m1} - \dot{g}_{01} - \dot{g}'_{01})}$$

$$3) \Gamma_{02}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m0}}{\partial x^2} + \frac{\partial g_{m2}}{\partial x^0} - \frac{\partial g_{02}}{\partial x^m} \right) =$$

$$= \underline{\frac{1}{2} g^{0m} (\dot{g}_{m2} - \dot{g}_{02} - \dot{g}'_{02})}$$

$$4) \Gamma_{03}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m0}}{\partial x^3} + \frac{\partial g_{m3}}{\partial x^0} - \frac{\partial g_{03}}{\partial x^m} \right) =$$

$$= \underline{\frac{1}{2} g^{0m} (\dot{g}_{m3} - \dot{g}_{03} - \dot{g}'_{03})}$$

$$5) \Gamma_{11}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m1}}{\partial x^1} + \frac{\partial g_{m1}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^m} \right) =$$

$$= \underline{\frac{1}{2} g^{0m} (\dot{g}'_{m1} + \dot{g}'_{m1} - \dot{g}_{11} - \dot{g}'_{11})}$$

$$6) \Gamma_{12}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m1}}{\partial x^2} + \frac{\partial g_{m2}}{\partial x^1} - \frac{\partial g_{12}}{\partial x^m} \right) =$$

$$= \underline{\frac{1}{2} g^{0m} (\dot{g}'_{m2} - \dot{g}_{12} - \dot{g}'_{12})}$$

$$7) \Gamma_{13}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m1}}{\partial x^3} + \frac{\partial g_{m3}}{\partial x^1} - \frac{\partial g_{13}}{\partial x^m} \right) =$$

$$= \frac{1}{2} g^{0m} \left(g'_{m3} - g'_{13} - g'_{13} \right)$$

$$8) \Gamma_{22}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m2}}{\partial x^2} + \frac{\partial g_{m2}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^m} \right) =$$

$$= \frac{1}{2} g^{0m} \left(-g'_{22} - g'_{22} \right)$$

$$9) \Gamma_{23}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m2}}{\partial x^3} + \frac{\partial g_{m3}}{\partial x^2} - \frac{\partial g_{23}}{\partial x^m} \right) =$$

$$= \frac{1}{2} g^{0m} \left(-g'_{23} - g'_{23} \right)$$

$$10) \Gamma_{33}^0 = \frac{1}{2} g^{0m} \left(\frac{\partial g_{m3}}{\partial x^3} + \frac{\partial g_{m3}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^m} \right) =$$

$$= \frac{1}{2} g^{0m} \left(-g'_{33} - g'_{33} \right)$$

$$11) \Gamma_{00}^1 = \frac{1}{2} g^{1m} \left(\frac{\partial g_{m0}}{\partial x^0} + \frac{\partial g_{m0}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^m} \right) =$$

$$= \frac{1}{2} g^{1m} \left(2g'_{m0} - g'_{00} - g'_{00} \right)$$

$$12) \Gamma_{01}^1 = \frac{1}{2} g^{1m} \left(\frac{\partial g_{m0}}{\partial x^1} + \frac{\partial g_{m1}}{\partial x^0} - \frac{\partial g_{01}}{\partial x^m} \right) =$$

$$= \frac{1}{2} \left(g^{1m} g'_{m0} + g^{1m} g'_{m1} - g^{10} g'_{01} - g^{11} g'_{01} \right)$$

$$13) \Gamma_{02}^1 = \frac{1}{2} g^{1m} \left(\frac{\partial g_{m0}}{\partial x^2} + \frac{\partial g_{m2}}{\partial x^0} - \dots \right) =$$

$$= \frac{1}{2} \left(g^{1m} g'_{m2} - \dots \right)$$

$$14) \Gamma_{03}^1 = \frac{1}{2} g^{1m} \left(\frac{\partial g_{m0}}{\partial x^3} + \frac{\partial g_{m3}}{\partial x^0} - \dots \right) =$$

$$= \frac{1}{2} \left(g^{1m} g'_{m3} - \dots \right)$$

$$15) \Gamma_{11}^1 = \frac{1}{2} g^{1m} \left(\frac{\partial g_{m1}}{\partial x^1} + \frac{\partial g_{m1}}{\partial x^1} - \dots \right) =$$

$$= \frac{1}{2} \left(2g^{1m} g'_{m1} - \dots \right)$$

$$16) \Gamma_{12}^1 = \frac{1}{2} g^{1m} \left(\frac{\partial g_{m2}}{\partial x^2} + \frac{\partial g_{m2}}{\partial x^1} - \dots \right) =$$

$$= \frac{1}{2} \left(g^{1m} g'_{m2} - \dots \right)$$

$$17) \Gamma_{13}^1 = \frac{1}{2} g^{1m} \left(\frac{\partial g_{m2}}{\partial x^3} + \frac{\partial g_{m3}}{\partial x^1} - \dots \right) =$$

$$= \frac{1}{2} \left(g^{1m} g'_{m3} - \dots \right)$$

$$21) \Gamma_{00}^2 = \frac{1}{2} g^{2m} \left(2 \frac{\partial g_{m0}}{\partial x^0} - \dots \right)$$

$$22) \Gamma_{01}^2 = \frac{1}{2} g^{2m} \left(\frac{\partial g_{m0}}{\partial x^1} + \frac{\partial g_{m1}}{\partial x^0} - \dots \right) =$$

$$= \frac{1}{2} \left(g^{2m} g'_{m0} + g^{2m} \dot{g}_{m1} - \dots \right)$$

$$23) \Gamma_{02}^2 = \frac{1}{2} g^{2m} \left(\frac{\partial g_{m0}}{\partial x^2} + \frac{\partial g_{m2}}{\partial x^0} - \dots \right) =$$

$$= \frac{1}{2} \left(g^{2m} \dot{g}_{m2} - \dots \right)$$

$$25) \Gamma_{11}^2 = \frac{1}{2} g^{2m} \left(2 \frac{\partial g_{m1}}{\partial x^1} - \dots \right)$$

$$26) \Gamma_{12}^2 = \frac{1}{2} g^{2m} \left(\frac{\partial g_{m2}}{\partial x^2} + \frac{\partial g_{m2}}{\partial x^1} - \dots \right)$$

R_{0123} , R_{0131} , R_{0112} , R_{0223} , R_{0231} , R_{0212} , R_{0323} , R_{0331} ,
 R_{2331} , R_{2312}

$$R_{123}^0 = \frac{\partial \Gamma_{31}^0}{\partial x^2} - \frac{\partial \Gamma_{21}^0}{\partial x^3} + \Gamma_{2\downarrow}^0 \Gamma_{31}^{\downarrow} - \Gamma_{3\downarrow}^0 \Gamma_{21}^{\downarrow}$$

$$R_{131}^0 = \frac{\partial \Gamma_{11}^0}{\partial x^3} - \frac{\partial \Gamma_{31}^0}{\partial x^1} + \Gamma_{3\downarrow}^0 \Gamma_{11}^{\downarrow} - \Gamma_{1\downarrow}^0 \Gamma_{31}^{\downarrow}$$

$$R_{112}^0 = \frac{\partial \Gamma_{21}^0}{\partial x^1} - \frac{\partial \Gamma_{11}^0}{\partial x^2} + \Gamma_{1\downarrow}^0 \Gamma_{21}^{\downarrow} - \Gamma_{2\downarrow}^0 \Gamma_{11}^{\downarrow}$$

$$R_{223}^0 = \frac{\partial \Gamma_{32}^0}{\partial x^2} - \frac{\partial \Gamma_{22}^0}{\partial x^3} + \Gamma_{2\downarrow}^0 \Gamma_{32}^{\downarrow} - \Gamma_{3\downarrow}^0 \Gamma_{22}^{\downarrow}$$

$$R_{231}^0 = \frac{\partial \Gamma_{12}^0}{\partial x^3} - \frac{\partial \Gamma_{32}^0}{\partial x^1} + \Gamma_{3\downarrow}^0 \Gamma_{12}^{\downarrow} - \Gamma_{1\downarrow}^0 \Gamma_{32}^{\downarrow}$$

$$R_{212}^0 = \frac{\partial \Gamma_{22}^0}{\partial x^1} - \frac{\partial \Gamma_{12}^0}{\partial x^2} + \Gamma_{1\downarrow}^0 \Gamma_{22}^{\downarrow} - \Gamma_{2\downarrow}^0 \Gamma_{12}^{\downarrow}$$

$$R_{323}^0 = \frac{\partial \Gamma_{33}^0}{\partial x^2} - \frac{\partial \Gamma_{23}^0}{\partial x^3} + \Gamma_{2\downarrow}^0 \Gamma_{33}^{\downarrow} - \Gamma_{3\downarrow}^0 \Gamma_{23}^{\downarrow}$$

$$R_{331}^0 = \frac{\partial \Gamma_{13}^0}{\partial x^3} - \frac{\partial \Gamma_{33}^0}{\partial x^1} + \Gamma_{3\downarrow}^0 \Gamma_{13}^{\downarrow} - \Gamma_{1\downarrow}^0 \Gamma_{33}^{\downarrow}$$

$$R_{331}^2 = \frac{\partial \Gamma_{13}^2}{\partial x^3} - \frac{\partial \Gamma_{33}^2}{\partial x^1} + \Gamma_{3\downarrow}^2 \Gamma_{13}^{\downarrow} - \Gamma_{1\downarrow}^2 \Gamma_{33}^{\downarrow}$$

$$R_{312}^2 = \frac{\partial \Gamma_{23}^2}{\partial x^1} - \frac{\partial \Gamma_{13}^2}{\partial x^2} + \Gamma_{1\downarrow}^2 \Gamma_{23}^{\downarrow} - \Gamma_{2\downarrow}^2 \Gamma_{13}^{\downarrow}$$

$$R_{123}^0 =$$

$$= \frac{1}{4} (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} g'_{02}) (g^{0m} g'_{m3} - g^{00} \dot{g}_{13} - g^{01} g'_{13}) +$$

$$+ \frac{1}{4} (g^{0m} g'_{m2} - g^{00} \dot{g}_{12} - g^{01} g'_{12}) (g^{1m} g'_{m3} - g^{10} \dot{g}_{13} - g^{11} g'_{13}) +$$

$$+ \frac{1}{4} (-g^{00} \dot{g}_{22} - g^{01} g'_{22}) (g^{2m} g'_{m3} - g^{20} \dot{g}_{13} - g^{21} g'_{13}) +$$

$$+ \frac{1}{4} (-g^{00} \dot{g}_{23} - g^{01} g'_{23}) (g^{3m} g'_{m3} - g^{30} \dot{g}_{13} - g^{31} g'_{13}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}_{m3} - g^{00} \dot{g}_{03} - g^{01} g'_{03}) (g^{0m} g'_{m2} - g^{00} \dot{g}_{12} - g^{01} g'_{12}) -$$

$$- \frac{1}{4} (g^{0m} g'_{m3} - g^{00} \dot{g}_{13} - g^{01} g'_{13}) (g^{0m} g'_{m2} - g^{00} \dot{g}_{12} - g^{01} g'_{12}) -$$

$$- \frac{1}{4} (-g^{00} \dot{g}_{23} - g^{01} g'_{23}) (g^{2m} g'_{m2} - g^{20} \dot{g}_{12} - g^{21} g'_{12}) -$$

$$- \frac{1}{4} (-g^{00} \dot{g}_{33} - g^{01} g'_{33}) (g^{3m} g'_{m2} - g^{30} \dot{g}_{12} - g^{31} g'_{12})$$

$$R_{131}^0 = -\frac{1}{2} \frac{\partial (g^{0m} g'_{m3} - g^{00} g'_{13} - g^{01} g'_{13})}{\partial x^1} +$$

(2)

$$\begin{aligned}
 & + \frac{1}{4} (g^{0m} g'_{m3} - g^{00} g'_{03} - g^{01} g'_{03}) (2g^{0m} g'_{m1} - g^{00} g'_{11} - g^{01} g'_{11}) + \\
 & + \frac{1}{4} (g^{0m} g'_{m3} - g^{00} g'_{13} - g^{01} g'_{13}) (2g^{1m} g'_{m1} - g^{11} g'_{11} - g^{11} g'_{11}) + \\
 & + \frac{1}{4} (-g^{00} g'_{23} - g^{01} g'_{23}) (2g^{2m} g'_{m1} - g^{20} g'_{11} - g^{21} g'_{11}) + \\
 & + \frac{1}{4} (-g^{00} g'_{33} - g^{01} g'_{33}) (2g^{3m} g'_{m1} - g^{30} g'_{11} - g^{31} g'_{11}) - \\
 & - \frac{1}{4} (g^{0m} g'_{m0} + g^{0m} g'_{m1} - g^{00} g'_{01} - g^{01} g'_{01}) (g^{0m} g'_{m3} - g^{00} g'_{13} - g^{01} g'_{13}) - \\
 & - \frac{1}{4} (2g^{0m} g'_{m1} - g^{00} g'_{11} - g^{01} g'_{11}) (g^{1m} g'_{m3} - g^{10} g'_{13} - g^{11} g'_{13}) - \\
 & - \frac{1}{4} (g^{0m} g'_{m2} - g^{00} g'_{12} - g^{01} g'_{12}) (g^{2m} g'_{m3} - g^{20} g'_{13} - g^{21} g'_{13}) - \\
 & - \frac{1}{4} (g^{0m} g'_{m3} - g^{00} g'_{13} - g^{01} g'_{13}) (g^{3m} g'_{m3} - g^{30} g'_{13} - g^{31} g'_{13})
 \end{aligned}$$

$$R_{112}^0 = \frac{1}{2} \frac{\partial (g^{0m} g'_{m2} - g^{00} g'_{12} - g^{01} g'_{12})}{\partial x^1} +$$

$$+ \frac{1}{4} (g^{0m} g'_{m0} + g^{0m} g'_{m1} - g^{00} g'_{01} - g^{01} g'_{01}) (g^{0m} g'_{m2} - g^{00} g'_{12} - g^{01} g'_{12}) +$$

$$+ \frac{1}{4} (2g^{0m} g'_{m1} - g^{00} g'_{11} - g^{01} g'_{11}) (g^{1m} g'_{m2} - g^{10} g'_{12} - g^{11} g'_{12}) +$$

$$+ \frac{1}{4} (g^{0m} g'_{m2} - g^{00} g'_{12} - g^{01} g'_{12}) (g^{2m} g'_{m2} - g^{20} g'_{12} - g^{21} g'_{12}) +$$

$$+ \frac{1}{4} (g^{0m} g'_{m3} - g^{00} g'_{13} - g^{01} g'_{13}) (g^{3m} g'_{m2} - g^{30} g'_{12} - g^{31} g'_{12}) -$$

$$- \frac{1}{4} (g^{0m} g'_{m2} - g^{00} g'_{02} - g^{01} g'_{02}) (2g^{0m} g'_{m1} - g^{00} g'_{11} - g^{01} g'_{11}) -$$

$$- \frac{1}{4} (g^{0m} g'_{m2} - g^{00} g'_{12} - g^{01} g'_{12}) (2g^{1m} g'_{m1} - g^{10} g'_{11} - g^{11} g'_{11}) -$$

$$- \frac{1}{4} (-g^{00} g'_{22} - g^{01} g'_{22}) (2g^{2m} g'_{m2} - g^{20} g'_{12} - g^{21} g'_{12}) -$$

$$- \frac{1}{4} (-g^{00} g'_{23} - g^{01} g'_{23}) (2g^{3m} g'_{m2} - g^{30} g'_{12} - g^{31} g'_{12})$$

$$R_{223}^0 =$$

(4)

$$\begin{aligned} &= \frac{1}{4} (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} \dot{g}'_{02}) (-g^{00} \dot{g}_{23} - g^{01} \dot{g}'_{23}) + \\ &+ \frac{1}{4} (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}_{12} - g^{01} \dot{g}'_{12}) (-g^{10} \dot{g}_{23} - g^{11} \dot{g}'_{23}) + \\ &+ \frac{1}{4} (-g^{00} \dot{g}_{22} - g^{01} \dot{g}'_{22}) (-g^{20} \dot{g}_{23} - g^{21} \dot{g}'_{23}) + \\ &+ \frac{1}{4} (-g^{00} \dot{g}_{23} - g^{01} \dot{g}'_{23}) (-g^{30} \dot{g}_{23} - g^{31} \dot{g}'_{23}) - \\ &- \frac{1}{4} (g^{0m} \dot{g}_{m3} - g^{00} \dot{g}_{03} - g^{01} \dot{g}'_{03}) (-g^{00} \dot{g}_{22} - g^{01} \dot{g}'_{22}) - \\ &- \frac{1}{4} (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}_{13} - g^{01} \dot{g}'_{13}) (-g^{10} \dot{g}_{22} - g^{11} \dot{g}'_{22}) - \\ &- \frac{1}{4} (-g^{00} \dot{g}_{33} - g^{01} \dot{g}'_{33}) (-g^{30} \dot{g}_{22} - g^{31} \dot{g}'_{22}) \end{aligned}$$

$$R_{231}^0 =$$

(5)

$$\begin{aligned} &= \frac{1}{4} (g^{0m} g_{m3} - g^{00} g_{03} - g^{01} g'_{03}) (g^{0m} g'_{m2} - g^{00} g_{12} - g^{01} g'_{12}) + \\ &+ \frac{1}{4} (g^{0m} g'_{m3} - g^{00} g_{13} - g^{01} g'_{13}) (g^{1m} g'_{m2} - g^{10} g_{12} - g^{11} g'_{12}) + \\ &+ \frac{1}{4} (-g^{00} g_{33} - g^{01} g'_{33}) (g^{3m} g'_{m2} - g^{30} g_{12} - g^{31} g'_{12}) \end{aligned}$$

$$R_{212}^0 = - \frac{\partial(-g^{00} \dot{g}_{22} - g^{01} g'_{22})}{\partial X^1} +$$

(6)

$$+ \frac{1}{4} (g^{0m} g'_{m0} + g^{0m} \dot{g}_{m1} - g^{00} \dot{g}_{01} - g^{01} g'_{01}) (-g^{00} \dot{g}_{22} - g^{01} g'_{22}) +$$

$$+ \frac{1}{4} (2g^{0m} g'_{m1} - g^{00} \dot{g}_{11} - g^{01} g'_{11}) (-g^{10} \dot{g}_{22} - g^{11} g'_{22}) +$$

$$+ \frac{1}{4} (g^{0m} g'_{m2} - g^{00} \dot{g}_{12} - g^{01} g'_{12}) (-g^{20} \dot{g}_{22} - g^{21} g'_{22}) +$$

$$+ \frac{1}{4} (g^{0m} g'_{m3} - g^{00} \dot{g}_{13} - g^{01} g'_{13}) (-g^{30} \dot{g}_{22} - g^{31} g'_{22}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} g'_{02}) (g^{0m} g'_{m2} - g^{00} \dot{g}_{12} - g^{01} g'_{12}) -$$

$$- \frac{1}{4} (g^{0m} g'_{m2} - g^{00} \dot{g}_{12} - g^{01} g'_{12}) (g^{1m} g'_{m2} - g^{10} \dot{g}_{12} - g^{11} g'_{12}) -$$

$$- \frac{1}{4} (-g^{00} \dot{g}_{22} - g^{01} g'_{22}) (g^{2m} g'_{m2} - g^{20} \dot{g}_{12} - g^{21} g'_{12})$$

(7)

$$R_{323}^0 =$$

$$= \frac{1}{4} (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} g'_{02}) (-g^{00} \dot{g}_{33} - g^{01} g'_{33}) +$$

$$+ \frac{1}{4} (g^{0m} g'_{m2} - g^{00} \dot{g}_{12} - g^{01} g'_{12}) (-g^{10} \dot{g}_{33} - g^{11} g'_{33}) +$$

$$+ \frac{1}{4} (-g^{00} \dot{g}_{22} - g^{01} g'_{22}) (-g^{20} \dot{g}_{33} - g^{21} g'_{33})$$

$$R_{331}^0 = - \frac{\partial(-g^{00}\dot{g}_{33} - g^{01}g'_{33})}{\partial X^1} +$$

(8)

$$+ \frac{1}{4} (g^{0m}\dot{g}_{m3} - g^{00}\dot{g}_{03} - g^{01}g'_{03}) (g^{0m}g'_{m3} - g^{00}\dot{g}_{13} - g^{01}g'_{13}) +$$

$$+ \frac{1}{4} (g^{0m}g'_{m3} - g^{00}\dot{g}_{13} - g^{01}g'_{13}) (g^{1m}g'_{m3} - g^{10}\dot{g}_{13} - g^{11}g'_{13}) +$$

$$+ \frac{1}{4} (-g^{00}\dot{g}_{33} - g^{01}g'_{33}) (g^{3m}g'_{m3} - g^{30}\dot{g}_{13} - g^{31}g'_{13}) -$$

$$- \frac{1}{4} (g^{0m}g'_{m0} + g^{0m}\dot{g}_{m1} - g^{00}\dot{g}_{01} - g^{01}g'_{01}) (-g^{00}\dot{g}_{33} - g^{01}g'_{33}) -$$

$$- \frac{1}{4} (2g^{0m}g'_{m1} - g^{00}\dot{g}_{11} - g^{01}g'_{11}) (-g^{10}\dot{g}_{33} - g^{11}g'_{33}) -$$

$$- \frac{1}{4} (g^{0m}g'_{m2} - g^{00}\dot{g}_{12} - g^{01}g'_{12}) (-g^{20}\dot{g}_{33} - g^{21}g'_{33}) -$$

$$- \frac{1}{4} (g^{0m}g'_{m3} - g^{00}\dot{g}_{13} - g^{01}g'_{13}) (-g^{30}\dot{g}_{33} - g^{31}g'_{33})$$

$$R_{331}^2 = - \frac{\partial (-g^{22} \dot{g}_{33} - g^{21} g'_{33})}{\partial x^1} +$$

(9)

$$+ \frac{1}{4} (g^{2m} \dot{g}_{m3} - g^{20} \dot{g}_{03} - g^{21} g'_{03}) (g^{0m} g'_{m3} - g^{00} \dot{g}_{13} - g^{01} g'_{13}) +$$

$$+ \frac{1}{4} (g^{2m} g'_{m3} - g^{20} \dot{g}_{13} - g^{21} g'_{13}) (g^{1m} g'_{m3} - g^{10} \dot{g}_{13} - g^{11} g'_{13}) +$$

$$+ \frac{1}{4} (-g^{20} \dot{g}_{33} - g^{21} g'_{33}) (g^{3m} g'_{m3} - g^{30} \dot{g}_{13} - g^{31} g'_{13}) -$$

$$- \frac{1}{4} (g^{2m} g'_{m0} + g^{2m} \dot{g}_{m1} - g^{20} \dot{g}_{01} - g^{21} g'_{01}) (-g^{00} \dot{g}_{33} - g^{01} g'_{33}) -$$

$$- \frac{1}{4} (2g^{2m} g'_{m1} - g^{20} \dot{g}_{11} - g^{21} g'_{11}) (-g^{10} \dot{g}_{33} - g^{11} g'_{33}) -$$

$$- \frac{1}{4} (g^{2m} g'_{m2} - g^{20} \dot{g}_{12} - g^{21} g'_{12}) (-g^{20} \dot{g}_{33} - g^{21} g'_{33}) -$$

$$- \frac{1}{4} (g^{2m} g'_{m3} - g^{20} \dot{g}_{13} - g^{21} g'_{13}) (-g^{30} \dot{g}_{33} - g^{31} g'_{33})$$

$$R_{312}^2 =$$

$$= -\frac{1}{4} (g^{2m} \dot{g}_{m2} - g^{20} \dot{g}_{02} - g^{21} \dot{g}'_{02}) (g^{0m} g'_{m3} - g^{00} \dot{g}_{13} - g^{01} \dot{g}'_{13}) -$$

$$- \frac{1}{4} (g^{2m} \dot{g}'_{m2} - g^{20} \dot{g}_{12} - g^{21} \dot{g}'_{12}) (g^{1m} g'_{m3} - g^{10} \dot{g}_{13} - g^{11} \dot{g}'_{13}) -$$

$$- \frac{1}{4} (-g^{20} \dot{g}_{22} - g^{21} \dot{g}'_{22}) (g^{2m} g'_{m3} - g^{20} \dot{g}_{13} - g^{21} \dot{g}'_{13})$$

$R_{0101}, R_{0102}, R_{0103}, R_{0202}, R_{0203}, R_{0303}, R_{1212}, R_{1213}, R_{1313}, R_{2323}$

$$R_{101}^0 = \frac{\partial \Gamma_{11}^0}{\partial x^0} - \frac{\partial \Gamma_{01}^0}{\partial x^1} + \Gamma_{00}^0 \Gamma_{11}^{\downarrow} - \Gamma_{10}^0 \Gamma_{01}^{\downarrow}$$

$$R_{102}^0 = \frac{\partial \Gamma_{21}^0}{\partial x^0} - \frac{\partial \Gamma_{01}^0}{\partial x^2} + \Gamma_{00}^0 \Gamma_{21}^{\downarrow} - \Gamma_{20}^0 \Gamma_{01}^{\downarrow}$$

$$R_{103}^0 = \frac{\partial \Gamma_{31}^0}{\partial x^0} - \frac{\partial \Gamma_{01}^0}{\partial x^3} + \Gamma_{00}^0 \Gamma_{31}^{\downarrow} - \Gamma_{30}^0 \Gamma_{01}^{\downarrow}$$

$$R_{202}^0 = \frac{\partial \Gamma_{22}^0}{\partial x^0} - \frac{\partial \Gamma_{02}^0}{\partial x^2} + \Gamma_{00}^0 \Gamma_{22}^{\downarrow} - \Gamma_{20}^0 \Gamma_{02}^{\downarrow}$$

$$R_{203}^0 = \frac{\partial \Gamma_{32}^0}{\partial x^0} - \frac{\partial \Gamma_{02}^0}{\partial x^3} + \Gamma_{00}^0 \Gamma_{32}^{\downarrow} - \Gamma_{30}^0 \Gamma_{02}^{\downarrow}$$

$$R_{303}^0 = \frac{\partial \Gamma_{33}^0}{\partial x^0} - \frac{\partial \Gamma_{03}^0}{\partial x^3} + \Gamma_{00}^0 \Gamma_{33}^{\downarrow} - \Gamma_{30}^0 \Gamma_{03}^{\downarrow}$$

$$R_{212}^1 = \frac{\partial \Gamma_{22}^1}{\partial x^1} - \frac{\partial \Gamma_{12}^1}{\partial x^2} + \Gamma_{10}^1 \Gamma_{22}^{\downarrow} - \Gamma_{21}^1 \Gamma_{12}^{\downarrow}$$

$$R_{213}^1 = \frac{\partial \Gamma_{32}^1}{\partial x^1} - \frac{\partial \Gamma_{12}^1}{\partial x^3} + \Gamma_{10}^1 \Gamma_{32}^{\downarrow} - \Gamma_{31}^1 \Gamma_{12}^{\downarrow}$$

$$R_{313}^1 = \frac{\partial \Gamma_{33}^1}{\partial x^1} - \frac{\partial \Gamma_{13}^1}{\partial x^3} + \Gamma_{10}^1 \Gamma_{33}^{\downarrow} - \Gamma_{31}^1 \Gamma_{13}^{\downarrow}$$

$$R_{323}^2 = \frac{\partial \Gamma_{33}^2}{\partial x^2} - \frac{\partial \Gamma_{23}^2}{\partial x^3} + \Gamma_{20}^2 \Gamma_{33}^{\downarrow} - \Gamma_{32}^2 \Gamma_{23}^{\downarrow}$$

$$1) \Gamma_{00}^0 \Gamma_{11}^0 + \Gamma_{01}^0 \Gamma_{11}^1 + \Gamma_{02}^0 \Gamma_{11}^2 + \Gamma_{03}^0 \Gamma_{11}^3 - \Gamma_{10}^0 \Gamma_{01}^0 - \Gamma_{11}^0 \Gamma_{01}^1 - \Gamma_{12}^0 \Gamma_{01}^2 - \Gamma_{13}^0 \Gamma_{01}^3 \quad (+)$$

$$2) \Gamma_{00}^0 \Gamma_{21}^0 + \Gamma_{01}^0 \Gamma_{21}^1 + \Gamma_{02}^0 \Gamma_{21}^2 + \Gamma_{03}^0 \Gamma_{21}^3 - \Gamma_{20}^0 \Gamma_{01}^0 - \Gamma_{21}^0 \Gamma_{01}^1 - \Gamma_{22}^0 \Gamma_{01}^2 - \Gamma_{23}^0 \Gamma_{01}^3 \quad (+) 0$$

$$3) \Gamma_{00}^0 \Gamma_{31}^0 + \Gamma_{01}^0 \Gamma_{31}^1 + \Gamma_{02}^0 \Gamma_{31}^2 + \Gamma_{03}^0 \Gamma_{31}^3 - \Gamma_{30}^0 \Gamma_{01}^0 - \Gamma_{31}^0 \Gamma_{01}^1 - \Gamma_{32}^0 \Gamma_{01}^2 - \Gamma_{33}^0 \Gamma_{01}^3 \quad (+) 0$$

$$4) \Gamma_{00}^0 \Gamma_{22}^0 + \Gamma_{01}^0 \Gamma_{22}^1 + \Gamma_{02}^0 \Gamma_{22}^2 + \Gamma_{03}^0 \Gamma_{22}^3 - \Gamma_{20}^0 \Gamma_{02}^0 - \Gamma_{21}^0 \Gamma_{02}^1 - \Gamma_{22}^0 \Gamma_{02}^2 - \Gamma_{23}^0 \Gamma_{02}^3 \quad (+)$$

$$5) \Gamma_{00}^0 \Gamma_{32}^0 + \Gamma_{01}^0 \Gamma_{32}^1 + \Gamma_{02}^0 \Gamma_{32}^2 + \Gamma_{03}^0 \Gamma_{32}^3 - \Gamma_{30}^0 \Gamma_{02}^0 - \Gamma_{31}^0 \Gamma_{02}^1 - \Gamma_{32}^0 \Gamma_{02}^2 - \Gamma_{33}^0 \Gamma_{02}^3 \quad (+) 0$$

$$6) \Gamma_{00}^0 \Gamma_{33}^0 + \Gamma_{01}^0 \Gamma_{33}^1 + \Gamma_{02}^0 \Gamma_{33}^2 + \Gamma_{03}^0 \Gamma_{33}^3 - \Gamma_{30}^0 \Gamma_{03}^0 - \Gamma_{31}^0 \Gamma_{03}^1 - \Gamma_{32}^0 \Gamma_{03}^2 - \Gamma_{33}^0 \Gamma_{03}^3 \quad (+)$$

$$7) \Gamma_{10}^1 \Gamma_{22}^0 + \Gamma_{11}^1 \Gamma_{22}^1 + \Gamma_{12}^1 \Gamma_{22}^2 + \Gamma_{13}^1 \Gamma_{22}^3 - \Gamma_{20}^1 \Gamma_{12}^0 - \Gamma_{21}^1 \Gamma_{12}^1 - \Gamma_{22}^1 \Gamma_{12}^2 - \Gamma_{23}^1 \Gamma_{12}^3 \quad (+)$$

$$8) \Gamma_{10}^1 \Gamma_{32}^0 + \Gamma_{11}^1 \Gamma_{32}^1 + \Gamma_{12}^1 \Gamma_{32}^2 + \Gamma_{13}^1 \Gamma_{32}^3 - \Gamma_{30}^1 \Gamma_{12}^0 - \Gamma_{31}^1 \Gamma_{12}^1 - \Gamma_{32}^1 \Gamma_{12}^2 - \Gamma_{33}^1 \Gamma_{12}^3 \quad (+) 0$$

$$9) \Gamma_{10}^1 \Gamma_{33}^0 + \Gamma_{11}^1 \Gamma_{33}^1 + \Gamma_{12}^1 \Gamma_{33}^2 + \Gamma_{13}^1 \Gamma_{33}^3 - \Gamma_{30}^1 \Gamma_{13}^0 - \Gamma_{31}^1 \Gamma_{13}^1 - \Gamma_{32}^1 \Gamma_{13}^2 - \Gamma_{33}^1 \Gamma_{13}^3 \quad (+)$$

$$10) \Gamma_{20}^2 \Gamma_{32}^0 + \Gamma_{21}^2 \Gamma_{32}^1 + \Gamma_{22}^2 \Gamma_{32}^2 + \Gamma_{23}^2 \Gamma_{32}^3 - \Gamma_{30}^2 \Gamma_{23}^0 - \Gamma_{31}^2 \Gamma_{23}^1 - \Gamma_{32}^2 \Gamma_{23}^2 - \Gamma_{33}^2 \Gamma_{23}^3 \quad (+) 0$$

$$R_{101}^0 = \frac{1}{2} \frac{\partial}{\partial x^0} (2g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{11} - g^{01} \dot{g}'_{12}) -$$

(1)

$$- \frac{1}{2} \frac{\partial}{\partial x^1} (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{02}) +$$

$$+ \frac{1}{4} (2g^{0m} \dot{g}'_{m0} - g^{00} \dot{g}'_{00} - g^{01} \dot{g}'_{01}) (2g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{11} - g^{01} \dot{g}'_{12}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{02}) (2g^{1m} \dot{g}'_{m1} - g^{10} \dot{g}'_{11} - g^{11} \dot{g}'_{12}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{02} - g^{01} \dot{g}'_{03}) (2g^{2m} \dot{g}'_{m1} - g^{20} \dot{g}'_{11} - g^{21} \dot{g}'_{12}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}'_{03} - g^{01} \dot{g}'_{03}) (2g^{3m} \dot{g}'_{m1} - g^{30} \dot{g}'_{11} - g^{31} \dot{g}'_{12}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{02}) (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{02}) -$$

$$- \frac{1}{4} (2g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{11} - g^{01} \dot{g}'_{12}) (g^{1m} \dot{g}'_{m0} + g^{1m} \dot{g}'_{m1} - g^{10} \dot{g}'_{01} - g^{11} \dot{g}'_{02}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{12} - g^{01} \dot{g}'_{12}) (g^{2m} \dot{g}'_{m0} + g^{2m} \dot{g}'_{m1} - g^{20} \dot{g}'_{01} - g^{21} \dot{g}'_{02}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}'_{13} - g^{01} \dot{g}'_{13}) (g^{3m} \dot{g}'_{m0} + g^{3m} \dot{g}'_{m1} - g^{30} \dot{g}'_{01} - g^{31} \dot{g}'_{02})$$

$$\frac{1}{4} (4g^{0m}g_{m0}g'^{0m}g'_{m1} - 2g^{0m}g_{m0}g^{00}g'_{11} - 2g^{0m}g_{m0}g^{01}g'_{11} - 2g^{00}g'_{00}g^{0m}g'_{m1} + g^{00}g'_{00}g^{00}g'_{11} + g^{00}g'_{00}g^{01}g'_{11} - 2g^{01}g'_{00}g^{0m}g'_{m1} + g^{01}g'_{00}g^{00}g'_{11} + g^{01}g'_{00}g^{01}g'_{11} + 2g^{0m}g'_{m0}g^{1m}g'_{m1} - g^{0m}g'_{m0}g^{10}g'_{11} - g^{0m}g'_{m0}g^{11}g'_{11} + 2g^{0m}g'_{m1}g^{1m}g'_{m1} - g^{0m}g'_{m1}g^{10}g'_{11} - g^{0m}g'_{m1}g^{11}g'_{11} - 2g^{00}g'_{01}g^{1m}g'_{m1} + g^{00}g'_{01}g^{10}g'_{11} + g^{00}g'_{01}g^{11}g'_{11} - 2g^{01}g'_{01}g^{1m}g'_{m1} + g^{01}g'_{01}g^{10}g'_{11} + g^{01}g'_{01}g^{11}g'_{11} +$$

$$+ 2g^{0m}g'_{m2}g^{2m}g'_{m1} - g^{0m}g'_{m2}g^{20}g'_{11} - g^{0m}g'_{m2}g^{21}g'_{11} - 2g^{00}g'_{02}g^{2m}g'_{m1} + g^{00}g'_{02}g^{20}g'_{11} + g^{00}g'_{02}g^{21}g'_{11} - 2g^{01}g'_{02}g^{2m}g'_{m1} + g^{01}g'_{02}g^{20}g'_{11} + g^{01}g'_{02}g^{21}g'_{11} +$$

$$+ 2g^{0m}g'_{m3}g^{3m}g'_{m1} - g^{0m}g'_{m3}g^{30}g'_{11} - g^{0m}g'_{m3}g^{31}g'_{11} - 2g^{00}g'_{03}g^{3m}g'_{m1} + g^{00}g'_{03}g^{30}g'_{11} + g^{00}g'_{03}g^{31}g'_{11} - 2g^{01}g'_{03}g^{3m}g'_{m1} + g^{01}g'_{03}g^{30}g'_{11} + g^{01}g'_{03}g^{31}g'_{11}$$

$$- g^{0m}g'_{m0}g^{0m}g'_{m0} + g^{0m}g'_{m0}g^{0m}g'_{m1} + g^{0m}g'_{m0}g^{00}g'_{01} + g^{0m}g'_{m0}g^{01}g'_{01} - g^{0m}g'_{m1}g^{0m}g'_{m0} - g^{0m}g'_{m1}g^{0m}g'_{m1} + g^{0m}g'_{m1}g^{00}g'_{01} + g^{0m}g'_{m1}g^{01}g'_{01} + g^{00}g'_{01}g^{0m}g'_{m0} + g^{00}g'_{01}g^{0m}g'_{m1} - g^{00}g'_{01}g^{00}g'_{01} - g^{00}g'_{01}g^{01}g'_{01} + g^{01}g'_{01}g^{0m}g'_{m0} + g^{01}g'_{01}g^{0m}g'_{m1} - g^{01}g'_{01}g^{00}g'_{01} - g^{01}g'_{01}g^{01}g'_{01} -$$

$$- 2g^{0m}g'_{m1}g^{1m}g'_{m0} - 2g^{0m}g'_{m1}g^{1m}g'_{m1} + 2g^{0m}g'_{m1}g^{10}g'_{01} + 2g^{0m}g'_{m1}g^{11}g'_{01} + g^{00}g'_{11}g^{1m}g'_{m0} + g^{00}g'_{11}g^{1m}g'_{m1} - g^{00}g'_{11}g^{10}g'_{01} - g^{00}g'_{11}g^{11}g'_{01} + g^{01}g'_{11}g^{1m}g'_{m0} + g^{01}g'_{11}g^{1m}g'_{m1} - g^{01}g'_{11}g^{10}g'_{01} - g^{01}g'_{11}g^{11}g'_{01} -$$

$$\begin{aligned}
 & -g^{0m} \dot{g}_{m2} g^{2m} g'_{m0} - g^{0m} \dot{g}_{m2} g^{2m} \dot{g}_{m1} + g^{0m} \dot{g}_{m2} g^{20} \ddot{g}_{01} + g^{0m} \dot{g}_{m2} g^{21} g'_{01} + \textcircled{1} \\
 & + g^{00} \dot{g}_{12} g^{2m} g'_{m0} + g^{00} \dot{g}_{12} g^{2m} \dot{g}_{m1} - g^{00} \dot{g}_{12} g^{20} \ddot{g}_{01} - g^{00} \dot{g}_{12} g^{21} g'_{01} + \\
 & + g^{01} g'_{12} g^{2m} g'_{m0} + g^{01} g'_{12} g^{2m} \dot{g}_{m1} - g^{01} g'_{12} g^{20} \ddot{g}_{01} - g^{01} g'_{12} g^{21} g'_{01} -
 \end{aligned}$$

$$\begin{aligned}
 & -g^{0m} g'_{m3} g^{3m} g'_{m0} - g^{0m} g'_{m3} g^{3m} \dot{g}_{m1} + g^{0m} g'_{m3} g^{30} \ddot{g}_{01} + g^{0m} g'_{m3} g^{31} g'_{01} + \\
 & + g^{00} \dot{g}_{13} g^{3m} g'_{m0} + g^{00} \dot{g}_{13} g^{3m} \dot{g}_{m1} - g^{00} \dot{g}_{13} g^{30} \ddot{g}_{01} - g^{00} \dot{g}_{13} g^{31} g'_{01} + \\
 & + g^{01} g'_{13} g^{3m} g'_{m0} + g^{01} g'_{13} g^{3m} \dot{g}_{m1} - g^{01} g'_{13} g^{30} \ddot{g}_{01} - g^{01} g'_{13} g^{31} g'_{01}
 \end{aligned}$$

$$R_{102}^0 = \frac{1}{2} \frac{\partial (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{12} - g^{01} \dot{g}'_{12})}{\partial x^0} + \quad (2)$$

$$+ \frac{1}{4} (2g^{0m} \dot{g}'_{m0} - g^{00} \dot{g}'_{00} - g^{01} \dot{g}'_{00}) (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{12} - g^{01} \dot{g}'_{12}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{01}) (g^{1m} \dot{g}'_{m2} - g^{10} \dot{g}'_{12} - g^{11} \dot{g}'_{12}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{02} - g^{01} \dot{g}'_{02}) (g^{2m} \dot{g}'_{m2} - g^{20} \dot{g}'_{12} - g^{21} \dot{g}'_{12}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}'_{03} - g^{01} \dot{g}'_{03}) (g^{3m} \dot{g}'_{m2} - g^{30} \dot{g}'_{12} - g^{31} \dot{g}'_{12}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{02} - g^{01} \dot{g}'_{02}) (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{01}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{12} - g^{01} \dot{g}'_{12}) (g^{1m} \dot{g}'_{m0} + g^{1m} \dot{g}'_{m1} - g^{10} \dot{g}'_{01} - g^{11} \dot{g}'_{01}) -$$

$$- \frac{1}{4} (-g^{00} \dot{g}'_{22} - g^{01} \dot{g}'_{22}) (g^{2m} \dot{g}'_{m0} + g^{2m} \dot{g}'_{m1} - g^{20} \dot{g}'_{01} - g^{21} \dot{g}'_{01}) -$$

$$- \frac{1}{4} (-g^{00} \dot{g}'_{23} - g^{01} \dot{g}'_{23}) (g^{3m} \dot{g}'_{m0} + g^{3m} \dot{g}'_{m1} - g^{30} \dot{g}'_{01} - g^{31} \dot{g}'_{01})$$

$$\frac{1}{4} (2g^{0m} \dot{g}_{m0} g^{0m} g'_{m2} - 2g^{0m} \dot{g}_{m0} g^{00} \dot{g}'_{12} - 2g^{0m} \dot{g}_{m0} g^{01} g'_{12} -$$

$$- g^{00} \dot{g}_{00} g^{0m} g'_{m2} + g^{00} \dot{g}_{00} g^{00} \dot{g}'_{12} + g^{00} \dot{g}_{00} g^{01} g'_{12} -$$

$$- g^{01} \dot{g}'_{00} g^{0m} g'_{m2} + g^{01} \dot{g}'_{00} g^{00} \dot{g}'_{12} + g^{01} \dot{g}'_{00} g^{01} g'_{12} +$$

$$+ g^{0m} \dot{g}'_{m0} g^{1m} g'_{m2} - g^{0m} \dot{g}'_{m0} g^{10} \dot{g}'_{12} - g^{0m} \dot{g}'_{m0} g^{11} g'_{12} +$$

$$+ g^{0m} \dot{g}'_{m1} g^{1m} g'_{m2} - g^{0m} \dot{g}'_{m1} g^{10} \dot{g}'_{12} - g^{0m} \dot{g}'_{m1} g^{11} g'_{12} -$$

$$- g^{00} \dot{g}'_{01} g^{1m} g'_{m2} + g^{00} \dot{g}'_{01} g^{10} \dot{g}'_{12} + g^{00} \dot{g}'_{01} g^{11} g'_{12} -$$

$$- g^{01} \dot{g}'_{01} g^{1m} g'_{m2} + g^{01} \dot{g}'_{01} g^{10} \dot{g}'_{12} + g^{01} \dot{g}'_{01} g^{11} g'_{12} +$$

$$+ g^{0m} \dot{g}'_{m2} g^{2m} g'_{m2} - g^{0m} \dot{g}'_{m2} g^{20} \dot{g}'_{12} - g^{0m} \dot{g}'_{m2} g^{21} g'_{12} -$$

$$- g^{00} \dot{g}'_{02} g^{2m} g'_{m2} + g^{00} \dot{g}'_{02} g^{20} \dot{g}'_{12} + g^{00} \dot{g}'_{02} g^{21} g'_{12} -$$

$$- g^{01} \dot{g}'_{02} g^{2m} g'_{m2} + g^{01} \dot{g}'_{02} g^{20} \dot{g}'_{12} + g^{01} \dot{g}'_{02} g^{21} g'_{12} +$$

$$+ g^{0m} \dot{g}'_{m3} g^{3m} g'_{m2} - g^{0m} \dot{g}'_{m3} g^{30} \dot{g}'_{12} - g^{0m} \dot{g}'_{m3} g^{31} g'_{12} -$$

$$- g^{00} \dot{g}'_{03} g^{3m} g'_{m2} + g^{00} \dot{g}'_{03} g^{30} \dot{g}'_{12} + g^{00} \dot{g}'_{03} g^{31} g'_{12} -$$

$$- g^{01} \dot{g}'_{03} g^{3m} g'_{m2} + g^{01} \dot{g}'_{03} g^{30} \dot{g}'_{12} + g^{01} \dot{g}'_{03} g^{31} g'_{12} -$$

$$- g^{0m} \dot{g}'_{m2} g^{0m} g'_{m0} - g^{0m} \dot{g}'_{m2} g^{0m} \dot{g}'_{m1} + g^{0m} \dot{g}'_{m2} g^{00} \dot{g}'_{01} + g^{0m} \dot{g}'_{m2} g^{01} \dot{g}'_{01} +$$

$$+ g^{00} \dot{g}'_{02} g^{0m} g'_{m0} + g^{00} \dot{g}'_{02} g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{02} g^{00} \dot{g}'_{01} - g^{00} \dot{g}'_{02} g^{01} \dot{g}'_{01} +$$

$$+ g^{01} \dot{g}'_{02} g^{0m} g'_{m0} + g^{01} \dot{g}'_{02} g^{0m} \dot{g}'_{m1} - g^{01} \dot{g}'_{02} g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{02} g^{01} \dot{g}'_{01} -$$

$$- g^{0m} \dot{g}'_{m2} g^{1m} g'_{m0} - g^{0m} \dot{g}'_{m2} g^{1m} \dot{g}'_{m1} + g^{0m} \dot{g}'_{m2} g^{10} \dot{g}'_{01} + g^{0m} \dot{g}'_{m2} g^{11} \dot{g}'_{01} +$$

$$+ g^{00} \dot{g}'_{12} g^{1m} g'_{m0} + g^{00} \dot{g}'_{12} g^{1m} \dot{g}'_{m1} - g^{00} \dot{g}'_{12} g^{10} \dot{g}'_{01} - g^{00} \dot{g}'_{12} g^{11} \dot{g}'_{01} +$$

$$+ g^{01} \dot{g}'_{12} g^{1m} g'_{m0} + g^{01} \dot{g}'_{12} g^{1m} \dot{g}'_{m1} - g^{01} \dot{g}'_{12} g^{10} \dot{g}'_{01} - g^{01} \dot{g}'_{12} g^{11} \dot{g}'_{01} +$$

$$+ g^{00} \dot{g}_{22} g^{2m} g'_{m0} + g^{00} \dot{g}_{22} g^{2m} \dot{g}_{m1} - g^{00} \dot{g}_{22} g^{20} \dot{g}_{01} - g^{00} \dot{g}_{22} g^{21} \dot{g}'_{01} + (2'')$$

$$+ g^{01} \dot{g}'_{22} g^{2m} g'_{m0} + g^{01} \dot{g}'_{22} g^{2m} \dot{g}_{m1} - g^{01} \dot{g}'_{22} g^{20} \dot{g}_{01} - g^{01} \dot{g}'_{22} g^{21} \dot{g}'_{01} +$$

~~$$+ g^{00} \dot{g}_{23} g^{3m} g'_{m0} + g^{00} \dot{g}_{23} g^{3m} \dot{g}_{m1} - g^{00} \dot{g}_{23} g^{30} \dot{g}_{01} - g^{00} \dot{g}_{23} g^{31} \dot{g}'_{01} +$$~~

~~$$+ g^{01} \dot{g}'_{23} g^{3m} g'_{m0} + g^{01} \dot{g}'_{23} g^{3m} \dot{g}_{m1} - g^{01} \dot{g}'_{23} g^{30} \dot{g}_{01} - g^{01} \dot{g}'_{23} g^{31} \dot{g}'_{01}$$~~

$$R_{103}^0 = \frac{1}{2} \frac{\partial (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}'_{13} - g^{01} \dot{g}'_{13})}{\partial x^0} +$$

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$$+ \frac{1}{4} (2g^{0m} \dot{g}'_{m0} - g^{00} \dot{g}'_{00} - g^{01} \dot{g}'_{00}) (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}'_{13} - g^{01} \dot{g}'_{13}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{01}) (g^{1m} \dot{g}'_{m3} - g^{10} \dot{g}'_{13} - g^{11} \dot{g}'_{13}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{02} - g^{01} \dot{g}'_{02}) (g^{2m} \dot{g}'_{m3} - g^{20} \dot{g}'_{13} - g^{21} \dot{g}'_{13}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}'_{03} - g^{01} \dot{g}'_{03}) (g^{3m} \dot{g}'_{m3} - g^{30} \dot{g}'_{13} - g^{31} \dot{g}'_{13}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}'_{03} - g^{01} \dot{g}'_{03}) (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{01}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}'_{13} - g^{01} \dot{g}'_{13}) (g^{1m} \dot{g}'_{m0} + g^{1m} \dot{g}'_{m1} - g^{10} \dot{g}'_{01} - g^{11} \dot{g}'_{01}) -$$

~~$$- \frac{1}{4} (-g^{00} \dot{g}'_{23} - g^{01} \dot{g}'_{23}) (g^{2m} \dot{g}'_{m0} + g^{2m} \dot{g}'_{m1} - g^{20} \dot{g}'_{01} - g^{21} \dot{g}'_{01}) -$$~~

$$- \frac{1}{4} (-g^{00} \dot{g}'_{33} - g^{01} \dot{g}'_{33}) (g^{3m} \dot{g}'_{m0} + g^{3m} \dot{g}'_{m1} - g^{30} \dot{g}'_{01} - g^{31} \dot{g}'_{01})$$

$$\frac{1}{4} (2g^{0m} \dot{g}_{m0} g^{0m} g'_{m3} - 2g^{0m} \dot{g}_{m0} g^{00} \dot{g}'_{13} - 2g^{0m} \dot{g}_{m0} g^{01} g'_{13} -$$

$$- g^{00} \dot{g}'_{00} g^{0m} g'_{m3} + g^{00} \dot{g}'_{00} g^{00} \dot{g}'_{13} + g^{00} \dot{g}'_{00} g^{01} g'_{13} -$$

$$- g^{01} \dot{g}'_{00} g^{0m} g'_{m3} + g^{01} \dot{g}'_{00} g^{00} \dot{g}'_{13} + g^{01} \dot{g}'_{00} g^{01} g'_{13} +$$

$$+ g^{0m} \dot{g}'_{m0} g^{1m} g'_{m3} - g^{0m} \dot{g}'_{m0} g^{10} \dot{g}'_{13} - g^{0m} \dot{g}'_{m0} g^{11} g'_{13} +$$

$$+ g^{0m} \dot{g}'_{m1} g^{1m} g'_{m3} - g^{0m} \dot{g}'_{m1} g^{10} \dot{g}'_{13} - g^{0m} \dot{g}'_{m1} g^{11} g'_{13} -$$

$$- g^{00} \dot{g}'_{01} g^{1m} g'_{m3} + g^{00} \dot{g}'_{01} g^{10} \dot{g}'_{13} + g^{00} \dot{g}'_{01} g^{11} g'_{13} -$$

$$- g^{01} \dot{g}'_{01} g^{1m} g'_{m3} + g^{01} \dot{g}'_{01} g^{10} \dot{g}'_{13} + g^{01} \dot{g}'_{01} g^{11} g'_{13} +$$

$$+ g^{0m} \dot{g}'_{m2} g^{2m} g'_{m3} - g^{0m} \dot{g}'_{m2} g^{20} \dot{g}'_{13} - g^{0m} \dot{g}'_{m2} g^{21} g'_{13} -$$

$$- g^{00} \dot{g}'_{02} g^{2m} g'_{m3} + g^{00} \dot{g}'_{02} g^{20} \dot{g}'_{13} - g^{00} \dot{g}'_{02} g^{21} g'_{13} -$$

$$- g^{01} \dot{g}'_{02} g^{2m} g'_{m3} + g^{01} \dot{g}'_{02} g^{20} \dot{g}'_{13} - g^{01} \dot{g}'_{02} g^{21} g'_{13} +$$

$$+ g^{0m} \dot{g}'_{m3} g^{3m} g'_{m3} - g^{0m} \dot{g}'_{m3} g^{30} \dot{g}'_{13} - g^{0m} \dot{g}'_{m3} g^{31} g'_{13} -$$

$$- g^{00} \dot{g}'_{03} g^{3m} g'_{m3} + g^{00} \dot{g}'_{03} g^{30} \dot{g}'_{13} + g^{00} \dot{g}'_{03} g^{31} g'_{13} -$$

$$- g^{01} \dot{g}'_{03} g^{3m} g'_{m3} + g^{01} \dot{g}'_{03} g^{30} \dot{g}'_{13} + g^{01} \dot{g}'_{03} g^{31} g'_{13} -$$

$$- g^{0m} \dot{g}'_{m3} g^{0m} g'_{m0} - g^{0m} \dot{g}'_{m3} g^{0m} g'_{m1} + g^{0m} \dot{g}'_{m3} g^{00} \dot{g}'_{01} + g^{0m} \dot{g}'_{m3} g^{01} g'_{01} +$$

$$+ g^{00} \dot{g}'_{03} g^{0m} g'_{m0} + g^{00} \dot{g}'_{03} g^{0m} g'_{m1} - g^{00} \dot{g}'_{03} g^{00} \dot{g}'_{01} - g^{00} \dot{g}'_{03} g^{01} g'_{01} +$$

$$+ g^{01} \dot{g}'_{03} g^{0m} g'_{m0} + g^{01} \dot{g}'_{03} g^{0m} g'_{m1} - g^{01} \dot{g}'_{03} g^{00} \dot{g}'_{01} - g^{01} \dot{g}'_{03} g^{01} g'_{01} -$$

$$- g^{0m} \dot{g}'_{m3} g^{1m} g'_{m0} - g^{0m} \dot{g}'_{m3} g^{1m} g'_{m1} + g^{0m} \dot{g}'_{m3} g^{10} \dot{g}'_{01} + g^{0m} \dot{g}'_{m3} g^{11} g'_{01} +$$

$$+ g^{00} \dot{g}'_{13} g^{1m} g'_{m0} + g^{00} \dot{g}'_{13} g^{1m} g'_{m1} - g^{00} \dot{g}'_{13} g^{10} \dot{g}'_{01} - g^{00} \dot{g}'_{13} g^{11} g'_{01} +$$

$$+ g^{01} \dot{g}'_{13} g^{1m} g'_{m0} + g^{01} \dot{g}'_{13} g^{1m} g'_{m1} - g^{01} \dot{g}'_{13} g^{10} \dot{g}'_{01} - g^{01} \dot{g}'_{13} g^{11} g'_{01} +$$

$$\begin{aligned}
 & + g^{00} g'_{23} g^{2m} g'_{m0} + g^{00} g'_{23} g^{2m} g'_{m1} - g^{00} g'_{23} g^{20} g'_{01} - g^{00} g'_{23} g^{21} g'_{01} \quad (3'') \\
 & + g^{01} g'_{23} g^{2m} g'_{m0} + g^{01} g'_{23} g^{2m} g'_{m1} - g^{01} g'_{23} g^{20} g'_{01} - g^{01} g'_{23} g^{21} g'_{01} +
 \end{aligned}$$

$$\begin{aligned}
 & + g^{00} g'_{33} g^{3m} g'_{m0} + g^{00} g'_{33} g^{3m} g'_{m1} - g^{00} g'_{33} g^{30} g'_{01} - g^{00} g'_{33} g^{31} g'_{01} + \\
 & + g^{01} g'_{33} g^{3m} g'_{m0} + g^{01} g'_{33} g^{3m} g'_{m1} - g^{01} g'_{33} g^{30} g'_{01} - g^{01} g'_{33} g^{31} g'_{01}
 \end{aligned}$$

$$R_{202}^0 = \frac{1}{2} \frac{\partial (-g^{00} \dot{g}_{22} - g^{01} g'_{22})}{\partial x^0} +$$

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$$\begin{aligned}
 & + \frac{1}{4} (2g^{0m} \dot{g}_{m0} - g^{00} \dot{g}_{00} - g^{01} g'_{00}) (-g^{00} \dot{g}_{22} - g^{01} g'_{22}) + \\
 & + \frac{1}{4} (g^{0m} g'_{m0} + g^{0m} \dot{g}_{m1} - g^{00} \dot{g}_{01} - g^{01} g'_{01}) (-g^{10} \dot{g}_{22} - g^{11} g'_{22}) + \\
 & + \frac{1}{4} (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} g'_{02}) (-g^{20} \dot{g}_{22} - g^{21} g'_{22}) + \\
 & + \frac{1}{4} (g^{0m} \dot{g}_{m3} - g^{00} \dot{g}_{03} - g^{01} g'_{03}) (-g^{30} \dot{g}_{22} - g^{31} g'_{22}) - \\
 & - \frac{1}{4} (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} g'_{02}) (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} g'_{02}) - \\
 & - \frac{1}{4} (g^{0m} g'_{m2} - g^{00} \dot{g}_{12} - g^{01} g'_{12}) (g^{1m} \dot{g}_{m2} - g^{10} \dot{g}_{02} - g^{11} g'_{02}) - \\
 & - \frac{1}{4} (-g^{00} \dot{g}_{22} - g^{01} g'_{22}) (g^{2m} \dot{g}_{m2} - g^{20} \dot{g}_{02} - g^{21} g'_{02}) - \\
 & - \frac{1}{4} (-g^{00} \dot{g}_{23} - g^{01} g'_{23}) (g^{3m} \dot{g}_{m2} - g^{30} \dot{g}_{02} - g^{31} g'_{02})
 \end{aligned}$$

$$\frac{1}{4} (2g^{0m} \dot{g}_{m0} g^{00} \dot{g}_{22} - 2g^{0m} \dot{g}_{m0} g^{01} \dot{g}'_{22} +$$

$$+ g^{00} \dot{g}'_{00} g^{00} \dot{g}_{22} + g^{00} \dot{g}'_{00} g^{01} \dot{g}'_{22} +$$

$$+ g^{01} \dot{g}'_{00} g^{00} \dot{g}_{22} + g^{01} \dot{g}'_{00} g^{01} \dot{g}'_{22} -$$

$$- g^{0m} \dot{g}'_{m0} g^{10} \dot{g}_{22} - g^{0m} \dot{g}'_{m0} g^{11} \dot{g}'_{22} -$$

$$- g^{0m} \dot{g}'_{m1} g^{10} \dot{g}_{22} - g^{0m} \dot{g}'_{m1} g^{11} \dot{g}'_{22} +$$

$$+ g^{00} \dot{g}'_{01} g^{10} \dot{g}_{22} + g^{00} \dot{g}'_{01} g^{11} \dot{g}'_{22} +$$

$$+ g^{01} \dot{g}'_{01} g^{10} \dot{g}_{22} + g^{01} \dot{g}'_{01} g^{11} \dot{g}'_{22} -$$

$$- g^{0m} \dot{g}'_{m2} g^{20} \dot{g}_{22} - g^{0m} \dot{g}'_{m2} g^{21} \dot{g}'_{22} +$$

$$+ g^{00} \dot{g}'_{02} g^{20} \dot{g}_{22} + g^{00} \dot{g}'_{02} g^{21} \dot{g}'_{22} +$$

$$+ g^{01} \dot{g}'_{02} g^{20} \dot{g}_{22} + g^{01} \dot{g}'_{02} g^{21} \dot{g}'_{22} -$$

$$- g^{0m} \dot{g}'_{m3} g^{30} \dot{g}_{22} - g^{0m} \dot{g}'_{m3} g^{31} \dot{g}'_{22} +$$

$$+ g^{00} \dot{g}'_{03} g^{30} \dot{g}_{22} + g^{00} \dot{g}'_{03} g^{31} \dot{g}'_{22} +$$

$$+ g^{01} \dot{g}'_{03} g^{30} \dot{g}_{22} + g^{01} \dot{g}'_{03} g^{31} \dot{g}'_{22} -$$

$$- g^{0m} \dot{g}'_{m2} g^{0m} \dot{g}'_{m2} + g^{0m} \dot{g}'_{m2} g^{00} \dot{g}'_{02} + g^{0m} \dot{g}'_{m2} g^{01} \dot{g}'_{02} +$$

$$+ g^{00} \dot{g}'_{02} g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{02} g^{00} \dot{g}'_{02} - g^{00} \dot{g}'_{02} g^{01} \dot{g}'_{02} +$$

$$+ g^{01} \dot{g}'_{02} g^{0m} \dot{g}'_{m2} - g^{01} \dot{g}'_{02} g^{00} \dot{g}'_{02} - g^{01} \dot{g}'_{02} g^{00} \dot{g}'_{02} -$$

$$- g^{0m} \dot{g}'_{m2} g^{1m} \dot{g}'_{m2} + g^{0m} \dot{g}'_{m2} g^{10} \dot{g}'_{02} + g^{0m} \dot{g}'_{m2} g^{11} \dot{g}'_{02} +$$

$$+ g^{00} \dot{g}'_{12} g^{1m} \dot{g}'_{m2} - g^{00} \dot{g}'_{12} g^{10} \dot{g}'_{02} - g^{00} \dot{g}'_{12} g^{11} \dot{g}'_{02} +$$

$$+ g^{01} \dot{g}'_{12} g^{1m} \dot{g}'_{m2} - g^{01} \dot{g}'_{12} g^{10} \dot{g}'_{02} - g^{01} \dot{g}'_{12} g^{11} \dot{g}'_{02} +$$

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$$+ g^{00} \dot{g}_{22} g^{2m} \dot{g}_{m2} - g^{00} \dot{g}_{22} g^{20} \dot{g}_{02} - g^{00} \dot{g}_{22} g^{21} \dot{g}'_{02} +$$

$$+ g^{01} \dot{g}'_{22} g^{2m} \dot{g}_{m2} - g^{01} \dot{g}'_{22} g^{20} \dot{g}_{02} - g^{01} \dot{g}_{22} g^{21} \dot{g}'_{02} +$$

~~$$+ g^{00} \dot{g}_{23} g^{3m} \dot{g}_{m2} - g^{00} \dot{g}_{23} g^{30} \dot{g}_{02} - g^{00} \dot{g}_{23} g^{31} \dot{g}'_{02} +$$~~

~~$$+ g^{01} \dot{g}'_{23} g^{3m} \dot{g}_{m2} - g^{01} \dot{g}'_{23} g^{30} \dot{g}_{02} - g^{01} \dot{g}'_{23} g^{31} \dot{g}'_{02}$$~~

$$R_{203}^0 = \frac{1}{2} \frac{\partial (-g^{00} \dot{g}_{23} - g^{01} \dot{g}'_{23})}{\partial x^0} +$$

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$$+ \frac{1}{4} (2g^{0m} \dot{g}_{m0} - g^{00} \dot{g}_{00} - g^{01} \dot{g}'_{00}) (-g^{00} \dot{g}_{23} - g^{01} \dot{g}'_{23}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}_{m0} + g^{0m} \dot{g}_{m1} - g^{00} \dot{g}_{01} - g^{01} \dot{g}'_{01}) (-g^{10} \dot{g}_{23} - g^{11} \dot{g}'_{23}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} \dot{g}'_{02}) (-g^{20} \dot{g}_{23} - g^{21} \dot{g}'_{23}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}_{m3} - g^{00} \dot{g}_{03} - g^{01} \dot{g}'_{03}) (-g^{30} \dot{g}_{23} - g^{31} \dot{g}'_{23}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}_{m3} - g^{00} \dot{g}_{03} - g^{01} \dot{g}'_{03}) (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} \dot{g}'_{02}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}_{13} - g^{01} \dot{g}'_{13}) (g^{1m} \dot{g}_{m2} - g^{10} \dot{g}_{02} - g^{11} \dot{g}'_{02}) -$$

$$- \frac{1}{4} (-g^{00} \dot{g}_{23} - g^{01} \dot{g}'_{23}) (g^{2m} \dot{g}_{m2} - g^{20} \dot{g}_{02} - g^{21} \dot{g}'_{02}) -$$

$$- \frac{1}{4} (-g^{00} \dot{g}_{33} - g^{01} \dot{g}'_{33}) (g^{3m} \dot{g}_{m2} - g^{30} \dot{g}_{02} - g^{31} \dot{g}'_{02})$$

~~$$\frac{1}{4} (2g^{0m} \dot{g}_{m0} g^{00} \dot{g}_{23} - 2g^{0m} \dot{g}_{m0} g^{11} g_{23} +$$

$$+ g^{00} \dot{g}_{00} g^{00} \dot{g}_{23} + g^{00} \dot{g}_{00} g^{01} \dot{g}_{23} +$$

$$+ g^{01} \dot{g}'_{00} g^{00} \dot{g}_{23} + g^{01} \dot{g}'_{00} g^{01} \dot{g}'_{23} -$$~~

~~$$- g^{0m} \dot{g}'_{m0} g^{10} \dot{g}_{23} - g^{0m} \dot{g}'_{m0} g^{11} \dot{g}'_{23} -$$

$$- g^{0m} \dot{g}'_{m1} g^{10} \dot{g}_{23} - g^{0m} \dot{g}'_{m1} g^{11} \dot{g}'_{23} +$$

$$+ g^{00} \dot{g}'_{01} g^{10} \dot{g}_{23} + g^{00} \dot{g}'_{01} g^{11} \dot{g}'_{23} +$$

$$+ g^{01} \dot{g}'_{01} g^{10} \dot{g}_{23} + g^{01} \dot{g}'_{01} g^{11} \dot{g}'_{23} -$$~~

~~$$- g^{0m} \dot{g}'_{m2} g^{20} \dot{g}_{23} - g^{0m} \dot{g}'_{m2} g^{21} \dot{g}'_{23} +$$

$$+ g^{00} \dot{g}'_{02} g^{20} \dot{g}_{23} + g^{00} \dot{g}'_{02} g^{21} \dot{g}'_{23} +$$

$$+ g^{01} \dot{g}'_{02} g^{20} \dot{g}_{23} + g^{01} \dot{g}'_{02} g^{21} \dot{g}'_{23} -$$~~

~~$$- g^{0m} \dot{g}'_{m3} g^{30} \dot{g}_{23} - g^{0m} \dot{g}'_{m3} g^{31} \dot{g}'_{23} +$$

$$+ g^{00} \dot{g}'_{03} g^{30} \dot{g}_{23} + g^{00} \dot{g}'_{03} g^{31} \dot{g}'_{23} +$$

$$+ g^{01} \dot{g}'_{03} g^{30} \dot{g}_{23} + g^{01} \dot{g}'_{03} g^{31} \dot{g}'_{23} -$$~~

$$- g^{0m} \dot{g}'_{m3} g^{0m} \dot{g}'_{m2} + g^{0m} \dot{g}'_{m3} g^{00} \dot{g}'_{02} + g^{0m} \dot{g}'_{m3} g^{01} \dot{g}'_{02} +$$

$$+ g^{00} \dot{g}'_{03} g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{03} g^{00} \dot{g}'_{02} - g^{00} \dot{g}'_{03} g^{01} \dot{g}'_{02} +$$

$$+ g^{01} \dot{g}'_{03} g^{0m} \dot{g}'_{m2} - g^{01} \dot{g}'_{03} g^{00} \dot{g}'_{02} - g^{01} \dot{g}'_{03} g^{01} \dot{g}'_{02} -$$

$$- g^{0m} \dot{g}'_{m3} g^{1m} \dot{g}'_{m2} + g^{0m} \dot{g}'_{m3} g^{10} \dot{g}'_{02} + g^{0m} \dot{g}'_{m3} g^{11} \dot{g}'_{02} +$$

$$+ g^{00} \dot{g}'_{13} g^{1m} \dot{g}'_{m2} - g^{00} \dot{g}'_{13} g^{10} \dot{g}'_{02} - g^{00} \dot{g}'_{13} g^{11} \dot{g}'_{02} +$$

$$+ g^{01} \dot{g}'_{13} g^{1m} \dot{g}'_{m2} - g^{01} \dot{g}'_{13} g^{10} \dot{g}'_{02} - g^{01} \dot{g}'_{13} g^{11} \dot{g}'_{02} +$$

~~+ g⁰⁰ g₂₃ g^{2m} g_{m2} - g⁰⁰ g₂₃ g²⁰ g₀₂ - g⁰⁰ g₂₃ g²¹ g₀₂ +~~

~~+ g⁰¹ g₂₃ g^{2m} g_{m2} - g⁰¹ g₂₃ g²⁰ g₀₂ - g⁰¹ g₂₃ g²¹ g₀₂ +~~

+ g⁰⁰ g₃₃ g^{3m} g_{m2} - g⁰⁰ g₃₃ g³⁰ g₀₂ - g⁰⁰ g₃₃ g³¹ g₀₂ +

+ g⁰¹ g₃₃ g^{3m} g_{m2} - g⁰¹ g₃₃ g³⁰ g₀₂ - g⁰¹ g₃₃ g³¹ g₀₂)

$$R_{303}^0 = \frac{1}{2} \frac{\partial (-g^{00} \dot{g}_{33} - g^{01} g'_{33})}{\partial x^0} + \quad (6)$$

$$+ \frac{1}{4} (2g^{0m} \dot{g}_{m0} - g^{00} \dot{g}_{00} - g^{01} g'_{00}) (-g^{00} \dot{g}_{33} - g^{01} g'_{33}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}_{m1} - g^{00} \dot{g}_{01} - g^{01} g'_{01}) (-g^{10} \dot{g}_{33} - g^{11} g'_{33}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}_{m2} - g^{00} \dot{g}_{02} - g^{01} g'_{02}) (-g^{20} \dot{g}_{33} - g^{21} g'_{33}) +$$

$$+ \frac{1}{4} (g^{0m} \dot{g}_{m3} - g^{00} \dot{g}_{03} - g^{01} g'_{03}) (-g^{30} \dot{g}_{33} - g^{31} g'_{33}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}_{m3} - g^{00} \dot{g}_{03} - g^{01} g'_{03}) (g^{0m} \dot{g}_{m3} - g^{00} \dot{g}_{03} - g^{01} g'_{03}) -$$

$$- \frac{1}{4} (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}_{13} - g^{01} g'_{13}) (g^{1m} \dot{g}_{m3} - g^{10} \dot{g}_{03} - g^{11} g'_{03}) -$$

~~$$- \frac{1}{4} (-g^{00} \dot{g}_{23} - g^{01} g'_{23}) (g^{2m} \dot{g}_{m3} - g^{20} \dot{g}_{03} - g^{21} g'_{03}) -$$~~

$$- \frac{1}{4} (-g^{00} \dot{g}_{33} - g^{01} g'_{33}) (g^{3m} \dot{g}_{m3} - g^{30} \dot{g}_{03} - g^{31} g'_{03})$$

(6')

$$\frac{1}{4} \left(-2g^{0m} \dot{g}_{m0} g^{00} \dot{g}_{33} - 2g^{0m} \dot{g}_{m0} g^{01} \dot{g}'_{33} + \right. \\ \left. + g^{00} \dot{g}_{00} g^{00} \dot{g}_{33} + g^{00} \dot{g}_{00} g^{01} \dot{g}'_{33} + \right. \\ \left. + g^{01} \dot{g}'_{00} g^{00} \dot{g}_{33} + g^{01} \dot{g}'_{00} g^{01} \dot{g}'_{33} - \right.$$

$$\left. - g^{0m} \dot{g}'_{m0} g^{10} \dot{g}_{33} - g^{0m} \dot{g}'_{m0} g^{11} \dot{g}'_{33} - \right. \\ \left. - g^{0m} \dot{g}_{m1} g^{10} \dot{g}_{33} - g^{0m} \dot{g}_{m1} g^{11} \dot{g}'_{33} + \right. \\ \left. + g^{00} \dot{g}_{01} g^{10} \dot{g}_{33} + g^{00} \dot{g}_{01} g^{11} \dot{g}'_{33} + \right. \\ \left. + g^{01} \dot{g}'_{01} g^{10} \dot{g}_{33} + g^{01} \dot{g}'_{01} g^{11} \dot{g}'_{33} - \right.$$

$$\left. - g^{0m} \dot{g}_{m2} g^{20} \dot{g}_{33} - g^{0m} \dot{g}_{m2} g^{21} \dot{g}'_{33} + \right. \\ \left. + g^{00} \dot{g}_{02} g^{20} \dot{g}_{33} + g^{00} \dot{g}_{02} g^{21} \dot{g}'_{33} + \right. \\ \left. + g^{01} \dot{g}'_{02} g^{20} \dot{g}_{33} + g^{01} \dot{g}'_{02} g^{21} \dot{g}'_{33} - \right.$$

$$\left. - g^{0m} \dot{g}_{m3} g^{30} \dot{g}_{33} - g^{0m} \dot{g}_{m3} g^{31} \dot{g}'_{33} + \right. \\ \left. + g^{00} \dot{g}_{03} g^{30} \dot{g}_{33} + g^{00} \dot{g}_{03} g^{31} \dot{g}'_{33} + \right. \\ \left. + g^{01} \dot{g}'_{03} g^{30} \dot{g}_{33} + g^{01} \dot{g}'_{03} g^{31} \dot{g}'_{33} - \right.$$

$$\left. - g^{0m} \dot{g}_{m3} g^{0m} \dot{g}_{m3} + g^{0m} \dot{g}_{m3} g^{00} \dot{g}_{03} + g^{0m} \dot{g}_{m3} g^{01} \dot{g}'_{03} + \right. \\ \left. + g^{00} \dot{g}_{03} g^{0m} \dot{g}_{m3} - g^{00} \dot{g}_{03} g^{00} \dot{g}_{03} - g^{00} \dot{g}_{03} g^{01} \dot{g}'_{03} + \right. \\ \left. + g^{01} \dot{g}'_{03} g^{0m} \dot{g}_{m3} - g^{01} \dot{g}'_{03} g^{00} \dot{g}_{03} - g^{01} \dot{g}'_{03} g^{01} \dot{g}'_{03} - \right.$$

$$\left. - g^{0m} \dot{g}'_{m3} g^{1m} \dot{g}_{m3} + g^{0m} \dot{g}'_{m3} g^{10} \dot{g}_{03} + g^{0m} \dot{g}'_{m3} g^{11} \dot{g}'_{03} + \right. \\ \left. + g^{00} \dot{g}_{13} g^{1m} \dot{g}_{m3} + g^{00} \dot{g}_{13} g^{10} \dot{g}_{03} - g^{00} \dot{g}_{13} g^{11} \dot{g}'_{03} + \right. \\ \left. + g^{01} \dot{g}'_{13} g^{1m} \dot{g}_{m3} - g^{01} \dot{g}'_{13} g^{10} \dot{g}_{03} - g^{01} \dot{g}'_{13} g^{11} \dot{g}'_{03} + \right.$$

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$$\begin{aligned} & + g^{00} \dot{g}_{23} g^{2m} \dot{g}_{m3} - g^{00} \dot{g}_{23} g^{20} \dot{g}_{03} - g^{00} \dot{g}_{23} g^{21} \dot{g}'_{03} + \\ & + g^{01} \dot{g}'_{23} g^{2m} \dot{g}_{m3} - g^{01} \dot{g}'_{23} g^{20} \dot{g}_{03} - g^{01} \dot{g}'_{23} g^{21} \dot{g}'_{03} + \end{aligned}$$

$$\begin{aligned} & + g^{00} \dot{g}_{33} g^{3m} \dot{g}_{m3} - g^{00} \dot{g}_{33} g^{30} \dot{g}_{03} - g^{00} \dot{g}_{33} g^{31} \dot{g}'_{03} + \\ & + g^{01} \dot{g}'_{33} g^{3m} \dot{g}_{m3} - g^{01} \dot{g}'_{33} g^{30} \dot{g}_{03} - g^{01} \dot{g}'_{33} g^{31} \dot{g}'_{03} \end{aligned}$$

$$R_{212}^1 = \frac{1}{2} \frac{\partial (-g^{10} \dot{g}_{22} - g^{11} g'_{22})}{\partial X^1} +$$

$$+ \frac{1}{4} (g^{1m} \dot{g}'_{m0} + g^{1m} \dot{g}'_{m1} - g^{10} \dot{g}'_{02} - g^{11} \dot{g}'_{12}) (-g^{00} \dot{g}'_{22} - g^{01} g'_{22}) +$$

$$+ \frac{1}{4} (2g^{1m} \dot{g}'_{m1} - g^{10} \dot{g}'_{11} - g^{11} \dot{g}'_{11}) (-g^{10} \dot{g}'_{22} - g^{11} g'_{22}) +$$

$$+ \frac{1}{4} (g^{1m} \dot{g}'_{m2} - g^{10} \dot{g}'_{12} - g^{11} \dot{g}'_{12}) (-g^{10} \dot{g}'_{22} - g^{11} g'_{22}) +$$

$$+ \frac{1}{4} (g^{1m} \dot{g}'_{m3} - g^{10} \dot{g}'_{13} - g^{11} \dot{g}'_{13}) (-g^{30} \dot{g}'_{22} - g^{31} g'_{22}) -$$

$$- \frac{1}{4} (g^{1m} \dot{g}'_{m2} - g^{10} \dot{g}'_{02} - g^{11} \dot{g}'_{02}) (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{22} - g^{01} g'_{22}) -$$

$$\left. \begin{aligned} & - \frac{1}{4} (-g^{10} \dot{g}'_{22} - g^{11} g'_{22}) (g^{2m} \dot{g}'_{m2} - g^{20} \dot{g}'_{12} - g^{21} g'_{12}) - \\ & - \frac{1}{4} (g^{1m} \dot{g}'_{m2} - g^{10} \dot{g}'_{12} - g^{11} g'_{12}) (g^{1m} \dot{g}'_{m2} - g^{10} \dot{g}'_{12} - g^{11} g'_{12}) - \end{aligned} \right\}$$

~~$$- \frac{1}{4} (-g^{10} \dot{g}'_{23} - g^{11} g'_{23}) (g^{3m} \dot{g}'_{m2} - g^{30} \dot{g}'_{12} - g^{31} g'_{12})$$~~

$$\frac{1}{4}(-g^{1m} g'_{m0} g^{00} j_{22} - g^{1m} g'_{m0} g^{01} g'_{22} -$$

$$- g^{1m} j_{m1} g^{00} j_{22} - g^{1m} j_{m1} g^{01} g'_{22} +$$

$$+ g^{10} j_{01} g^{00} j_{22} + g^{10} j_{01} g^{01} g'_{22} +$$

$$+ g^{11} j'_{01} g^{00} j_{22} + g^{11} j'_{01} g^{01} g'_{22} -$$

$$- 2g^{1m} g'_{m1} g^{10} j_{22} - 2g^{1m} g'_{m1} g^{21} g'_{22} +$$

$$+ g^{10} j_{11} g^{10} j_{22} + g^{10} j_{11} g^{11} g'_{22} +$$

$$+ g^{11} j'_{11} g^{10} j_{22} + g^{11} j'_{11} g^{11} g'_{22} -$$

$$- g^{1m} g'_{m2} g^{20} j_{22} - g^{1m} g'_{m2} g^{21} g'_{22} +$$

$$+ g^{10} j_{12} g^{20} j_{22} + g^{10} j_{12} g^{21} g'_{22} +$$

$$+ g^{11} j'_{12} g^{20} j_{22} + g^{11} j'_{12} g^{21} g'_{22} -$$

$$- g^{1m} g'_{m3} g^{30} j_{22} - g^{1m} g'_{m3} g^{31} g'_{22} +$$

$$+ g^{10} j_{13} g^{30} j_{22} + g^{10} j_{13} g^{31} g'_{22} +$$

$$+ g^{11} j'_{13} g^{30} j_{22} + g^{11} j'_{13} g^{31} g'_{22} -$$

$$- g^{1m} j_{m2} g^{0m} g'_{m2} + g^{1m} j_{m2} g^{00} j_{12} + g^{1m} j_{m2} g^{01} g'_{12} +$$

$$+ g^{10} j_{02} g^{0m} g'_{m2} - g^{10} j_{02} g^{00} j_{12} - g^{10} j_{02} g^{01} g'_{12} +$$

$$+ g^{11} j'_{02} g^{0m} g'_{m2} - g^{11} j'_{02} g^{00} j_{12} - g^{11} j'_{02} g^{01} g'_{12} +$$

$$+ g^{10} j_{22} g^{2m} g'_{m2} - g^{10} j_{22} g^{20} j_{12} - g^{10} j_{22} g^{21} g'_{12} +$$

$$+ g^{11} j'_{22} g^{2m} g'_{m2} - g^{11} j'_{22} g^{20} j_{12} - g^{11} j'_{22} g^{21} g'_{12} -$$

$$\begin{aligned}
& -g^{1m} g'_{m2} g^{1m} g'_{m2} + g^{1m} g'_{m2} g^{10} g'_{12} + g^{1m} g'_{m2} g^{11} g'_{12} + \\
& + g^{10} g'_{12} g^{1m} g'_{m2} - g^{10} g'_{12} g^{10} g'_{12} - g^{10} g'_{12} g^{11} g'_{12} + \\
& + g^{11} g'_{12} g^{1m} g'_{m2} - g^{11} g'_{12} g^{10} g'_{12} - g^{11} g'_{12} g^{11} g'_{12} +
\end{aligned}$$

~~$$\begin{aligned}
& + g^{10} g'_{23} g^{3m} g'_{m2} - g^{10} g'_{23} g^{30} g'_{12} - g^{10} g'_{23} g^{11} g'_{12} + \\
& + g^{11} g'_{23} g^{3m} g'_{m2} - g^{11} g'_{23} g^{30} g'_{12} - g^{11} g'_{23} g^{11} g'_{12}
\end{aligned}$$~~

$$R_{213}^1 = \frac{1}{2} \frac{\partial (-g^{10} \dot{g}_{23} - g^{11} \dot{g}'_{23})}{\partial x^1} +$$

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$$+ \frac{1}{4} (g^{0m} \dot{g}'_{m0} + g^{0m} \dot{g}'_{m1} - g^{00} \dot{g}'_{02} - g^{01} \dot{g}'_{02}) (-g^{00} \dot{g}'_{23} - g^{01} \dot{g}'_{23}) +$$

$$+ \frac{1}{4} (2g^{1m} \dot{g}'_{m1} - g^{10} \dot{g}'_{11} - g^{11} \dot{g}'_{11}) (-g^{00} \dot{g}'_{23} - g^{01} \dot{g}'_{23}) +$$

$$+ \frac{1}{4} (g^{1m} \dot{g}'_{m2} - g^{10} \dot{g}'_{12} - g^{11} \dot{g}'_{12}) (-g^{20} \dot{g}'_{23} - g^{21} \dot{g}'_{23}) +$$

$$+ \frac{1}{4} (g^{1m} \dot{g}'_{m3} - g^{10} \dot{g}'_{13} - g^{11} \dot{g}'_{13}) (-g^{30} \dot{g}'_{23} - g^{31} \dot{g}'_{23}) -$$

$$- \frac{1}{4} (g^{1m} \dot{g}'_{m3} - g^{10} \dot{g}'_{03} - g^{11} \dot{g}'_{03}) (g^{0m} \dot{g}'_{m2} - g^{00} \dot{g}'_{12} - g^{01} \dot{g}'_{12}) -$$

$$- \frac{1}{4} (g^{1m} \dot{g}'_{m3} - g^{10} \dot{g}'_{13} - g^{11} \dot{g}'_{13}) (g^{1m} \dot{g}'_{m2} - g^{10} \dot{g}'_{12} - g^{11} \dot{g}'_{12}) -$$

$$- \frac{1}{4} (-g^{10} \dot{g}'_{23} - g^{11} \dot{g}'_{23}) (g^{2m} \dot{g}'_{m2} - g^{20} \dot{g}'_{12} - g^{21} \dot{g}'_{12}) -$$

$$- \frac{1}{4} (-g^{10} \dot{g}'_{33} - g^{11} \dot{g}'_{33}) (g^{3m} \dot{g}'_{m2} - g^{30} \dot{g}'_{12} - g^{31} \dot{g}'_{12})$$

$$\frac{1}{4} \left(\cancel{g^{0m} g'_{m0} g^{00} j_{23} - g^{0m} g'_{m0} g^{01} j'_{23} -} \right. \\
\left. \cancel{- g^{0m} j_{m0} g^{00} j_{23} - g^{0m} j_{m0} g^{01} j'_{23} +} \right. \\
\left. \cancel{+ g^{00} j_{01} g^{00} j_{23} + g^{00} j_{01} g^{01} j'_{23} +} \right. \\
\left. \cancel{+ g^{01} j'_{01} g^{00} j_{23} + g^{01} j'_{01} g^{01} j'_{23} -} \right.$$

$$\left. \cancel{- 2g^{1m} g'_{m1} g^{00} j_{23} - 2g^{1m} g'_{m1} g^{01} j'_{23} +} \right. \\
\left. \cancel{+ g^{10} j_{11} g^{00} j_{23} + g^{10} j_{11} g^{01} j'_{23} +} \right. \\
\left. \cancel{+ g^{11} j'_{11} g^{00} j_{23} + g^{11} j'_{11} g^{01} j'_{23} -} \right.$$

$$\left. \cancel{- g^{1m} g'_{m2} g^{20} j_{23} - g^{1m} g'_{m2} g^{21} j'_{23} +} \right. \\
\left. \cancel{+ g^{10} j_{12} g^{20} j_{23} + g^{10} j_{12} g^{21} j'_{23} +} \right. \\
\left. \cancel{+ g^{11} j'_{12} g^{20} j_{23} + g^{11} j'_{12} g^{21} j'_{23} -} \right.$$

$$\left. \cancel{- g^{1m} g'_{m3} g^{30} j_{23} - g^{1m} g'_{m3} g^{31} j'_{23} +} \right. \\
\left. \cancel{+ g^{10} j_{13} g^{30} j_{23} + g^{10} j_{13} g^{31} j'_{23} +} \right. \\
\left. \cancel{+ g^{11} j'_{13} g^{30} j_{23} + g^{11} j'_{13} g^{31} j'_{23} -} \right.$$

$$\left. - g^{1m} j_{m3} g^{0m} g'_{m2} + g^{1m} j_{m3} g^{00} j_{12} + g^{1m} j_{m3} g^{01} g'_{12} + \right. \\
\left. + g^{10} j_{03} g^{0m} g'_{m2} - g^{10} j_{03} g^{00} j_{12} - g^{10} j_{03} g^{01} g'_{12} + \right. \\
\left. + g^{11} g'_{03} g^{0m} g'_{m2} - g^{11} g'_{03} g^{00} j_{12} - g^{11} g'_{03} g^{01} g'_{12} - \right.$$

$$\left. - g^{1m} g'_{m3} g^{1m} g'_{m2} + g^{1m} g'_{m3} g^{10} j_{12} + g^{1m} g'_{m3} g^{11} g'_{12} + \right. \\
\left. + g^{10} j_{13} g^{1m} g'_{m2} - g^{10} j_{13} g^{10} j_{12} - g^{10} j_{13} g^{11} g'_{12} + \right. \\
\left. + g^{11} g'_{13} g^{1m} g'_{m2} - g^{11} g'_{13} g^{10} j_{12} - g^{11} g'_{13} g^{11} g'_{12} + \right.$$

~~$$+ g^{10} j_{23} g^{2m} g'_{m2} - g^{10} j_{23} g^{20} j_{12} - g^{10} j_{23} g^{21} g'_{12} +$$~~

~~$$+ g^{11} g'_{23} g^{2m} g'_{m2} - g^{11} g'_{23} g^{20} j_{12} - g^{11} g'_{23} g^{21} g'_{12} +$$~~

$$+ g^{10} j_{33} g^{3m} g'_{m2} - g^{10} j_{33} g^{30} j_{12} - g^{10} j_{33} g^{31} g'_{12} +$$

$$+ g^{11} g'_{33} g^{3m} g'_{m2} - g^{11} g'_{33} g^{30} j_{12} - g^{11} g'_{33} g^{31} g'_{12})$$

$$R_{313}^1 = \frac{1}{2} \frac{\partial (-g^{10} \dot{g}_{33} - g^{11} g'_{33})}{\partial x^1} + \quad (9)$$

$$+ \frac{1}{4} (g^{1m} \dot{g}'_{m0} + g^{1m} \dot{g}'_{m1} - g^{10} \dot{g}'_{01} - g^{11} g'_{01}) (-g^{00} \dot{g}'_{33} - g^{01} g'_{33}) +$$

$$+ \frac{1}{4} (2g^{1m} \dot{g}'_{m2} - g^{10} \dot{g}'_{11} - g^{11} g'_{11}) (-g^{10} \dot{g}'_{23} - g^{11} g'_{23}) +$$

$$+ \frac{1}{4} (g^{1m} \dot{g}'_{m2} - g^{10} \dot{g}'_{12} - g^{11} g'_{12}) (-g^{20} \dot{g}'_{33} - g^{21} g'_{33}) +$$

$$+ \frac{1}{4} (g^{1m} \dot{g}'_{m3} - g^{10} \dot{g}'_{13} - g^{11} g'_{13}) (-g^{30} \dot{g}'_{33} - g^{31} g'_{33}) -$$

$$- \frac{1}{4} (g^{1m} \dot{g}'_{m3} - g^{10} \dot{g}'_{03} - g^{11} g'_{03}) (g^{0m} \dot{g}'_{m3} - g^{00} \dot{g}'_{13} - g^{01} g'_{13}) -$$

$$- \frac{1}{4} (g^{1m} \dot{g}'_{m3} - g^{10} \dot{g}'_{13} - g^{11} g'_{13}) (g^{1m} \dot{g}'_{m3} - g^{20} \dot{g}'_{13} - g^{21} g'_{13}) -$$

~~$$- \frac{1}{4} (-g^{10} \dot{g}'_{23} - g^{11} g'_{23}) (g^{2m} \dot{g}'_{m3} - g^{20} \dot{g}'_{13} - g^{21} g'_{13}) -$$~~

$$- \frac{1}{4} (-g^{10} \dot{g}'_{33} - g^{11} g'_{33}) (g^{3m} \dot{g}'_{m3} - g^{30} \dot{g}'_{13} - g^{31} g'_{13})$$

(g')

$$\frac{1}{4}(-g^{1m} g'_{m0} g^{00} g'_{33} - g^{1m} g'_{m0} g^{01} g'_{33} -$$

$$- g^{1m} g'_{m1} g^{00} g'_{33} - g^{1m} g'_{m1} g^{01} g'_{33} +$$

$$+ g^{10} g'_{01} g^{00} g'_{33} + g^{10} g'_{01} g^{01} g'_{33} +$$

$$+ g^{11} g'_{01} g^{00} g'_{33} + g^{11} g'_{01} g^{01} g'_{33} -$$

$$- 2g^{1m} g'_{m1} g^{10} g'_{33} - 2g^{1m} g'_{m1} g^{11} g'_{33} +$$

$$+ g^{10} g'_{11} g^{10} g'_{33} + g^{10} g'_{11} g^{11} g'_{33} +$$

$$+ g^{11} g'_{11} g^{10} g'_{33} + g^{11} g'_{11} g^{11} g'_{33} -$$

$$- g^{1m} g'_{m2} g^{20} g'_{33} - g^{1m} g'_{m2} g^{21} g'_{33} +$$

$$+ g^{10} g'_{12} g^{20} g'_{33} + g^{10} g'_{12} g^{21} g'_{33} +$$

$$+ g^{11} g'_{12} g^{20} g'_{33} + g^{11} g'_{12} g^{21} g'_{33} -$$

$$- g^{1m} g'_{m3} g^{30} g'_{33} - g^{1m} g'_{m3} g^{31} g'_{33} +$$

$$+ g^{10} g'_{13} g^{30} g'_{33} + g^{10} g'_{13} g^{31} g'_{33} +$$

$$+ g^{11} g'_{13} g^{30} g'_{33} + g^{11} g'_{13} g^{31} g'_{33} -$$

$$- g^{1m} g'_{m3} g^{0m} g'_{m3} + g^{1m} g'_{m3} g^{00} g'_{13} + g^{1m} g'_{m3} g^{01} g'_{13} +$$

$$+ g^{10} g'_{03} g^{0m} g'_{m3} - g^{10} g'_{03} g^{00} g'_{13} - g^{10} g'_{03} g^{01} g'_{13} +$$

$$+ g^{11} g'_{03} g^{0m} g'_{m3} - g^{11} g'_{03} g^{00} g'_{13} - g^{11} g'_{03} g^{01} g'_{13} -$$

$$- g^{1m} g'_{m3} g^{1m} g'_{m3} + g^{1m} g'_{m3} g^{10} g'_{13} + g^{1m} g'_{m3} g^{11} g'_{13} +$$

$$+ g^{10} g'_{13} g^{1m} g'_{m3} - g^{10} g'_{13} g^{10} g'_{13} - g^{10} g'_{13} g^{11} g'_{13} +$$

$$+ g^{11} g'_{13} g^{1m} g'_{m3} - g^{11} g'_{13} g^{10} g'_{13} - g^{11} g'_{13} g^{11} g'_{13} +$$

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$$+ g^{10} g_{23} g^{2m} g'_{m3} - g^{10} g_{23} g^{20} g'_{13} - g^{10} g_{23} g^{21} g'_{13} +$$

$$+ g^{11} g'_{23} g^{2m} g'_{m3} - g^{11} g'_{23} g^{20} g'_{13} - g^{11} g'_{23} g^{21} g'_{13} +$$

$$+ g^{10} g'_{33} g^{3m} g'_{m3} - g^{10} g'_{33} g^{30} g'_{13} - g^{10} g'_{33} g^{31} g'_{13} +$$

$$+ g^{11} g'_{33} g^{3m} g'_{m3} - g^{11} g'_{33} g^{30} g'_{13} - g^{11} g'_{33} g^{31} g'_{13})$$

$$R_{323}^2 = 0$$

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~~$$\begin{aligned} &= \frac{1}{4} (g^{2m} \dot{g}_{m2} - g^{20} \dot{g}_{02} - g^{21} \dot{g}'_{02}) (-g^{00} \dot{g}_{23} - g^{01} \dot{g}'_{23}) + \\ &+ \frac{1}{4} (g^{2m} \dot{g}'_{m2} - g^{20} \dot{g}_{12} - g^{21} \dot{g}'_{12}) (-g^{10} \dot{g}_{23} - g^{11} \dot{g}'_{23}) + \\ &+ \frac{1}{4} (-g^{20} \dot{g}_{22} - g^{21} \dot{g}'_{22}) (-g^{20} \dot{g}_{23} - g^{21} \dot{g}'_{23}) + \\ &+ \frac{1}{4} (-g^{20} \dot{g}_{23} - g^{21} \dot{g}'_{23}) (-g^{30} \dot{g}_{23} - g^{31} \dot{g}'_{23}) - \\ &- \frac{1}{4} (g^{2m} \dot{g}_{m3} - g^{20} \dot{g}_{03} - g^{21} \dot{g}'_{03}) (-g^{00} \dot{g}_{23} - g^{01} \dot{g}'_{23}) - \\ &- \frac{1}{4} (g^{2m} \dot{g}'_{m3} - g^{20} \dot{g}_{13} - g^{21} \dot{g}'_{13}) (-g^{10} \dot{g}_{23} - g^{11} \dot{g}'_{23}) - \\ &- \frac{1}{4} (-g^{20} \dot{g}_{23} - g^{21} \dot{g}'_{23}) (-g^{20} \dot{g}_{23} - g^{21} \dot{g}'_{23}) - \\ &- \frac{1}{4} (-g^{20} \dot{g}_{33} - g^{21} \dot{g}'_{33}) (-g^{30} \dot{g}_{23} - g^{31} \dot{g}'_{23}) \end{aligned}$$~~

$$\begin{aligned}
& \frac{1}{4} \left(-g^{2m} j_{m2} g^{00} j_{23} - g^{2m} j_{m2} g^{01} g'_{23} + \right. \\
& + g^{20} j_{02} g^{00} j_{23} + g^{20} j_{02} g^{01} g'_{23} + \\
& + g^{21} j_{02} g^{00} j_{23} + g^{21} j'_{02} g^{01} g'_{23} - \\
& - g^{2m} j'_{m2} g^{10} j_{23} - g^{2m} j'_{m2} g^{11} g'_{23} + \\
& + g^{20} j_{12} g^{10} j_{23} + g^{20} j_{12} g^{11} g'_{23} + \\
& + g^{21} j'_{12} g^{10} j_{23} + g^{21} j'_{12} g^{11} g'_{23} + \\
& + g^{20} j_{22} g^{20} j_{23} + g^{20} j_{22} g^{21} g'_{23} + \\
& + g^{21} g'_{22} g^{20} j_{23} + g^{21} g'_{22} g^{21} g'_{23} + \\
& + g^{20} j_{23} g^{30} j_{23} + g^{20} j_{23} g^{31} g'_{23} + \\
& + g^{21} g'_{23} g^{30} j_{23} + g^{21} g'_{23} g^{31} g'_{23} + \\
& + g^{2m} j_{m3} g^{00} j_{23} + g^{2m} j_{m3} g^{01} g'_{23} - \\
& - g^{20} j_{03} g^{00} j_{23} - g^{20} j_{03} g^{01} g'_{23} - \\
& - g^{21} g'_{03} g^{00} j_{23} - g^{21} g'_{03} g^{01} g'_{23} + \\
& + g^{2m} j'_{m3} g^{10} j_{23} + g^{2m} j'_{m3} g^{11} g'_{23} - \\
& - g^{20} j_{13} g^{10} j_{23} - g^{20} j_{13} g^{11} g'_{23} - \\
& - g^{21} g'_{13} g^{10} j_{23} - g^{21} g'_{13} g^{11} g'_{23} - \left. \right)
\end{aligned}$$

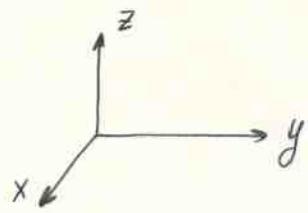
~~$$\begin{aligned}
 & - g^{20} g_{23} g^{20} g_{23} - g^{20} g_{23} g^{21} g'_{23} - \\
 & - g^{21} g'_{23} g^{20} g_{23} - g^{21} g'_{23} g^{21} g'_{23} - \\
 & - g^{20} g_{33} g^{30} g_{23} - g^{20} g_{33} g^{31} g'_{23} - \\
 & - g^{21} g'_{33} g^{30} g_{23} - g^{21} g'_{33} g^{31} g'_{23})
 \end{aligned}$$~~

$$g_{23} = 0$$

$$g_{02} = g_{03} = 0$$

$$g_{12} = g_{13} = 0$$

метрика декартова пространства
 неизменяется при замене
 $y \rightarrow -y,$
 $z \rightarrow -z.$



$$ds^2 = \alpha \cdot dt^2 + 2\beta \cdot dt dx + \gamma \cdot dx^2 + \omega \cdot (dy^2 + dz^2).$$

$$\begin{cases} \alpha = \alpha(t, x) \\ \vdots \\ \omega = \omega(t, x) \end{cases}$$

$$g_{00} \rightarrow \alpha$$

$$g_{01} \rightarrow \beta$$

$$g_{11} \rightarrow \gamma$$

$$g_{22} \rightarrow \omega$$

неинвариантно

$$g_{33} \rightarrow \omega$$

используя табличку сверху с учетом подбора.

Конрадов А. А., Демисов
 Рен. теория грав. (g_{ij})

"Математические теории гравитации"
 грав. Конрадов. М. 1985.

$$1) \Gamma_{00}^0 = \frac{1}{2} (2g^{01} \dot{g}_{10} - \cancel{2g^{00} \dot{g}_{00}} - \cancel{g^{01} g'_{00}} + 2g^{00} \dot{g}_{00})$$

$$2) \Gamma_{01}^0 = \frac{1}{2} (g^{01} g'_{10} + g^{01} \dot{g}_{11} - g^{00} \dot{g}_{01} - \cancel{g^{01} g'_{01}} + \underbrace{g^{00} g'_{00} + g^{00} \dot{g}_{01}})$$

$$3) \Gamma_{02}^0 = \cancel{\frac{1}{2} (2g^{01} \dot{g}_{10} - \dots)} = 0$$

$$4) \Gamma_{03}^0 = 0$$

$$5) \Gamma_{11}^0 = \frac{1}{2} (2g^{00} g'_{01} - g^{00} \dot{g}_{11} - g^{01} g'_{11})$$

$$6) \Gamma_{12}^0 = 0$$

$$7) \Gamma_{13}^0 = 0$$

$$8) \Gamma_{22}^0 = \frac{1}{2} (-g^{00} \dot{g}_{22} - g^{01} g'_{22})$$

$$9) \Gamma_{23}^0 = 0$$

$$10) \Gamma_{33}^0 = \frac{1}{2} (-g^{00} \dot{g}_{33} - g^{01} g'_{33})$$

$$11) \Gamma_{00}^1 = \frac{1}{2} (2g^{10} \dot{g}_{00} - g^{10} \dot{g}_{00} - \cancel{g^{11} g'_{00}} + 2g^{11} \dot{g}_{10})$$

$$12) \Gamma_{01}^1 = \frac{1}{2} (g^{10} g'_{00} + g^{11} g'_{10} + \cancel{g^{10} \dot{g}_{01}} + g^{11} \dot{g}_{11} - \cancel{g^{10} \dot{g}_{01}} - \cancel{g^{11} g'_{01}})$$

$$13) \Gamma_{02}^1 = 0$$

$$14) \Gamma_{03}^1 = 0$$

$$15) \Gamma_{11}^1 = \frac{1}{2} (2g^{10} g'_{01} + \cancel{2g^{11} g'_{11}} - g^{10} \dot{g}_{11} - \cancel{g^{11} g'_{11}})$$

$$16) \Gamma_{12}^1 = 0$$

$$17) \Gamma_{13}^1 = 0$$

$$18) \Gamma_{22}^1 = \frac{1}{2} (g^{10} \dot{g}_{22} - g^{11} g'_{22})$$

$$19) \Gamma_{23}^1 = 0$$

$$20) \Gamma_{33}^1 = \frac{1}{2} (-g^{10} \dot{g}_{33} - g^{11} g'_{33})$$

$$(21) \Gamma_{00}^2 = 0$$

$$(22) \Gamma_{01}^2 = 0$$

$$(23) \Gamma_{02}^2 = \frac{1}{2} g^{22} \dot{g}_{22}$$

$$(24) \Gamma_{03}^2 = 0$$

$$(25) \Gamma_{11}^2 = 0$$

$$(26) \Gamma_{12}^2 = \frac{1}{2} g^{22} \dot{g}_{22}$$

$$(27) \Gamma_{13}^2 = 0$$

$$(28) \Gamma_{22}^2 = 0$$

$$(29) \Gamma_{23}^2 = 0$$

$$(30) \Gamma_{33}^2 = 0$$

$$(31) \Gamma_{00}^3 = 0$$

$$(32) \Gamma_{01}^3 = 0$$

$$(33) \Gamma_{02}^3 = \frac{1}{2} g^{33} \dot{g}_{33}$$

$$(34) \Gamma_{03}^3 = 0$$

$$(35) \Gamma_{11}^3 = 0$$

$$(36) \Gamma_{12}^3 = \frac{1}{2} g^{33} \dot{g}_{33}$$

$$(37) \Gamma_{13}^3 = 0$$

$$(38) \Gamma_{22}^3 = 0$$

$$(39) \Gamma_{23}^3 = 0$$

$$(40) \Gamma_{33}^3 = 0$$

~~$$\frac{1}{2}(\alpha\alpha + 2\beta\beta - \alpha\alpha - \beta\beta)$$~~

(I)

$$\exists g_{em} = \begin{bmatrix} \alpha & \beta & 0 & 0 \\ \beta & \gamma & 0 & 0 \\ 0 & 0 & \omega & 0 \\ 0 & 0 & 0 & \omega \end{bmatrix} = \left[\begin{array}{cc|cc} \alpha & \beta & 0 & 0 \\ \beta & \gamma & & \\ \hline 0 & 0 & \omega & 0 \\ 0 & 0 & 0 & \omega \end{array} \right]$$

$\exists g^{em}$ - обратный к g_{em} .

Найдем g^{em} .

$$g_{em} g^{mk} = \delta_e^k \quad - \text{константные обратности тензоров.}$$

Рассмотрим это:

$$\left(\begin{array}{cc|cc} \alpha & \beta & 0 & 0 \\ \beta & \gamma & & \\ \hline 0 & 0 & \omega & 0 \\ 0 & 0 & 0 & \omega \end{array} \right) \left(\begin{array}{cc|cc} x & y & 0 & 0 \\ y & z & u & 0 \\ \hline 0 & 0 & u & 0 \end{array} \right) = \left(\begin{array}{ccc} 1 & 1 & 0 \\ & 1 & 1 \\ 0 & & 1 \end{array} \right)$$

Хотим искать в таком виде.

Переименовав матрицы:

~~$$\left(\begin{array}{cc|cc} \alpha x + \beta y & \beta y + \gamma z & 0 & 0 \\ \beta x + \gamma y & \gamma y + \delta z & u & 0 \\ \hline 0 & 0 & u & \omega \end{array} \right)$$~~

$$\left(\begin{array}{cc|cc} (\alpha x + \beta y) & (\beta y + \gamma z) & & 0 \\ (\beta x + \gamma y) & (\gamma y + \delta z) & & \\ \hline & & \omega u & 0 \\ 0 & & 0 & \omega u \end{array} \right) = \left(\begin{array}{ccc} 1 & & 0 \\ & 1 & 1 \\ 0 & & 1 \end{array} \right)$$

$$\begin{cases} \alpha x + \beta y = 1 \\ \beta y + \gamma z = 0 \\ \beta x + \gamma y = 0 \\ \gamma y + \delta z = 1 \\ \omega u = 1 \end{cases}$$

Решаем систему:

$$x = \frac{1 - \beta y}{\alpha}$$

$$x = -\frac{\gamma y}{\beta}$$

$$y = -\frac{\beta z}{\alpha}$$

$$y = \frac{1 - \gamma z}{\beta}$$

$$\frac{1 - \beta y}{\alpha} = -\frac{\gamma y}{\beta}$$

$$\beta(1 - \beta y) = -\gamma \alpha y$$

$$\beta - \beta^2 y + \gamma \alpha y = 0$$

$$y = -\frac{\beta}{\alpha \gamma - \beta^2}$$

$$-\frac{\beta z}{\alpha} = \frac{1 - \gamma z}{\beta}$$

$$-\beta^2 z = \alpha - \alpha \gamma z$$

$$\alpha \gamma z - \beta^2 z = \alpha$$

$$z = \frac{\alpha}{\alpha \gamma - \beta^2}$$

$$x = -\frac{\gamma}{\beta} \left(-\frac{\beta}{\alpha \gamma - \beta^2} \right)$$

$$x = \frac{\gamma}{\alpha \gamma - \beta^2}$$

$$u = \frac{1}{\omega}$$

~~y_{em}~~

$$g^{em} = \left[\begin{array}{cc|cc} \frac{\gamma}{\alpha \gamma - \beta^2} & -\frac{\beta}{\alpha \gamma - \beta^2} & & \\ -\frac{\beta}{\alpha \gamma - \beta^2} & \frac{\alpha}{\alpha \gamma - \beta^2} & & 0 \\ \hline & & \frac{1}{\omega} & 0 \\ & & 0 & \frac{1}{\omega} \end{array} \right]$$

$$\left\{ \begin{array}{l} g_{00} = \alpha \\ g_{01} = \beta \\ g_{11} = \gamma \\ g_{22} = \omega \\ g_{33} = \omega \end{array} \right. \left\{ \begin{array}{l} g^{00} = \frac{\gamma}{2\gamma - \beta^2} \\ g^{01} = -\frac{\beta}{2\gamma - \beta^2} \\ g^{11} = \frac{\alpha}{2\gamma - \beta^2} \\ g^{22} = \frac{1}{\omega} \\ g^{33} = \frac{1}{\omega} \end{array} \right.$$

$$1) \Gamma_{00}^0 = \frac{1}{2} \left(-2 \frac{\beta \dot{\beta}}{2\gamma - \beta^2} + \frac{\beta \alpha'}{2\gamma - \beta^2} + \frac{\gamma \dot{\alpha}}{2\gamma - \beta^2} \right)$$

$$2) \Gamma_{01}^0 = \frac{1}{2} \left(\frac{\gamma \alpha'}{2\gamma - \beta^2} - \frac{\beta \dot{\beta}'}{2\gamma - \beta^2} + \frac{\gamma \dot{\beta}}{2\gamma - \beta^2} + \frac{\beta \dot{\gamma}}{2\gamma - \beta^2} - \frac{\gamma \dot{\beta}}{2\gamma - \beta^2} + \frac{\beta \dot{\beta}'}{2\gamma - \beta^2} \right)$$

$$5) \Gamma_{11}^0 = \frac{1}{2} \left(2 \frac{\gamma \beta'}{2\gamma - \beta^2} - \frac{\gamma \dot{\gamma}}{2\gamma - \beta^2} + \frac{\beta \gamma'}{2\gamma - \beta^2} \right)$$

$$8) \Gamma_{22}^0 = \frac{1}{2} \left(-\frac{\gamma \dot{\omega}}{2\gamma - \beta^2} + \frac{\beta \omega'}{2\gamma - \beta^2} \right) = \Gamma_{33}^0 \leftarrow 10)$$

$$11) \Gamma_{00}^1 = \frac{1}{2} \left(\frac{\beta \dot{\alpha}}{2\gamma - \beta^2} + 2 \frac{\alpha \dot{\beta}}{2\gamma - \beta^2} + \frac{\beta \dot{\gamma}}{2\gamma - \beta^2} - \frac{\alpha \alpha'}{2\gamma - \beta^2} \right)$$

$$12) \Gamma_{01}^1 = \frac{1}{2} \left(-\frac{\beta \alpha'}{2\gamma - \beta^2} + \frac{\alpha \dot{\gamma}}{2\gamma - \beta^2} \right)$$

$$15) \Gamma_{11}^1 = \frac{1}{2} \left(-2 \frac{\beta \beta'}{2\gamma - \beta^2} + \frac{\alpha \gamma'}{2\gamma - \beta^2} + \frac{\beta \dot{\gamma}}{2\gamma - \beta^2} \right)$$

$$18) \Gamma_{22}^1 = \frac{1}{2} \left(-\frac{\beta \dot{\omega}}{2\gamma - \beta^2} - \frac{\alpha \omega'}{2\gamma - \beta^2} \right) = \Gamma_{33}^1 \leftarrow 20)$$

$$23) \Gamma_{02}^2 = \frac{1}{2} \frac{\dot{\omega}}{\omega} = \Gamma_{03}^3 \leftarrow 34)$$

$$26) \Gamma_{12}^2 = \frac{1}{2} \frac{\omega'}{\omega} = \Gamma_{13}^3 \leftarrow 32)$$

$$R_{\beta\alpha} = \frac{\partial \Gamma_{\beta\alpha}^{\sigma}}{\partial x^{\sigma}} - \frac{\partial \Gamma_{\sigma\alpha}^{\beta}}{\partial x^{\beta}} + \Gamma_{\sigma\gamma}^{\sigma} \Gamma_{\beta\alpha}^{\gamma} - \Gamma_{\beta\gamma}^{\sigma} \Gamma_{\sigma\alpha}^{\gamma}$$

$$R_{00}, R_{01}, R_{02}, R_{03},$$

$$R_{11}, R_{12}, R_{13},$$

$$R_{22}, R_{23},$$

$$R_{33}.$$

$$1) R_{02} = \frac{\partial \Gamma_{20}^{\sigma}}{\partial x^{\sigma}} - \frac{\partial \Gamma_{\sigma 0}^2}{\partial x^2} + \Gamma_{\sigma\gamma}^{\sigma} \Gamma_{20}^{\gamma} - \Gamma_{2\gamma}^{\sigma} \Gamma_{\sigma 0}^{\gamma}$$

$$\Gamma_{\sigma 2}^{\sigma} \Gamma_{02}^2 = 0, \quad \Gamma_{2\gamma}^{\sigma} \Gamma_{\sigma 0}^{\gamma} =$$

$$= \Gamma_{2\gamma}^{\sigma} \Gamma_{00}^{\gamma} + \Gamma_{2\gamma}^1 \Gamma_{20}^{\gamma} + \Gamma_{2\gamma}^2 \Gamma_{20}^{\gamma} + \Gamma_{2\gamma}^3 \Gamma_{30}^{\gamma} = 0$$

$$R_{02} = 0$$

$$2) R_{03} = 0$$

$$\Gamma_{\sigma 3}^{\sigma} \Gamma_{30}^3 = 0$$

$$\Gamma_{3\gamma}^{\sigma} \Gamma_{00}^{\gamma} + \Gamma_{3\gamma}^1 \Gamma_{20}^{\gamma} + \Gamma_{3\gamma}^2 \Gamma_{20}^{\gamma} + \Gamma_{3\gamma}^3 \Gamma_{30}^{\gamma}$$

$$3) R_{12} = \frac{\partial \Gamma_{21}^{\sigma}}{\partial x^{\sigma}} - \frac{\partial \Gamma_{\sigma 1}^2}{\partial x^2} + \Gamma_{\sigma\gamma}^{\sigma} \Gamma_{21}^{\gamma} - \Gamma_{2\gamma}^{\sigma} \Gamma_{\sigma 1}^{\gamma}$$

$$\Gamma_{2\gamma}^{\sigma} \Gamma_{\sigma 1}^{\gamma} = \Gamma_{2\gamma}^0 \Gamma_{01}^{\gamma} + \Gamma_{2\gamma}^1 \Gamma_{21}^{\gamma} + \Gamma_{2\gamma}^2 \Gamma_{21}^{\gamma} + \Gamma_{2\gamma}^3 \Gamma_{31}^{\gamma}$$

$$R_{12} = 0$$

$$4) R_{13} = \frac{\partial \Gamma_{31}^{\sigma}}{\partial x^{\sigma}} \Big|_{\sigma=3} - \frac{\partial \Gamma_{\sigma 1}^{\sigma}}{\partial x^3} + \frac{\Gamma_{\sigma \nu}^{\sigma} \Gamma_{31}^{\nu}}{\nu=3} - \frac{\Gamma_{3\nu}^{\sigma} \Gamma_{\sigma 1}^{\nu}}{0}$$

$$R_{13} = 0$$

$$5) R_{23} = \frac{\partial \Gamma_{32}^{\sigma}}{\partial x^{\sigma}} - \frac{\partial \Gamma_{\sigma 2}^{\sigma}}{\partial x^3} + \frac{\Gamma_{\sigma \nu}^{\sigma} \Gamma_{32}^{\nu}}{\nu=3} - \Gamma_{3\nu}^{\sigma} \Gamma_{\sigma 2}^{\nu}$$

$$\Gamma_{3\nu}^{\sigma} \Gamma_{\sigma 2}^{\nu} = \frac{\Gamma_{3\nu}^0 \Gamma_{02}^{\nu}}{0} + \frac{\Gamma_{3\nu}^1 \Gamma_{12}^{\nu}}{0} + \frac{\Gamma_{3\nu}^2 \Gamma_{22}^{\nu}}{0} + \cancel{\Gamma_{3\nu}^3 \Gamma_{32}^{\nu}}$$

$$R_{23} = 0$$

~~$$6) R_{33} = \frac{\partial \Gamma_{33}^{\sigma}}{\partial x^{\sigma}} - \frac{\partial \Gamma_{\sigma 3}^{\sigma}}{\partial x^3} + \frac{\Gamma_{\sigma \nu}^{\sigma} \Gamma_{33}^{\nu}}{\nu=0;1} - \frac{\Gamma_{3\nu}^{\sigma} \Gamma_{\sigma 3}^{\nu}}{\nu=0;1,3} =$$~~

~~$$= \frac{\partial \Gamma_{33}^0}{\partial t} + \frac{\partial \Gamma_{33}^1}{\partial x} + \cancel{\Gamma_{00}^0 \Gamma_{33}^0} + \Gamma_{20}^1 \Gamma_{33}^0 + \Gamma_{01}^0 \Gamma_{33}^1 + \Gamma_{21}^1 \Gamma_{33}^1 -$$~~

~~$$- \Gamma_{30}^3 \Gamma_{33}^0 - \Gamma_{33}^0 \Gamma_{03}^3 - \Gamma_{33}^1 \Gamma_{13}^3$$~~

~~$$7) R_{00} = \frac{\partial \Gamma_{00}^{\sigma}}{\partial x^{\sigma}} - \frac{\partial \Gamma_{\sigma 0}^{\sigma}}{\partial x^0} + \frac{\Gamma_{\sigma \nu}^{\sigma} \Gamma_{00}^{\nu}}{\nu=0;1} - \frac{\Gamma_{0\nu}^{\sigma} \Gamma_{\sigma 0}^{\nu}}{\nu=0;1} =$$~~

~~$$= \frac{\partial \Gamma_{00}^0}{\partial t} + \frac{\partial \Gamma_{00}^1}{\partial x} - \frac{\partial \Gamma_{00}^0}{\partial t} - \frac{\partial \Gamma_{10}^1}{\partial t} - \frac{\partial \Gamma_{20}^2}{\partial t} - \frac{\partial \Gamma_{30}^3}{\partial t} +$$~~

~~$$+ \frac{\Gamma_{00}^0 \Gamma_{00}^0}{\Gamma_{00}^0 \Gamma_{00}^0} + \cancel{\Gamma_{10}^1 \Gamma_{00}^1} + \Gamma_{02}^0 \Gamma_{00}^1 + \Gamma_{11}^1 \Gamma_{00}^1 -$$~~

~~$$- \frac{\Gamma_{00}^0 \Gamma_{00}^0}{\Gamma_{00}^0 \Gamma_{00}^0} - \cancel{\Gamma_{10}^1 \Gamma_{10}^1} - \cancel{\Gamma_{02}^0 \Gamma_{00}^1} - \Gamma_{01}^1 \Gamma_{10}^1 -$$~~

~~$$- \Gamma_{02}^2 \Gamma_{20}^2 - \Gamma_{03}^3 \Gamma_{30}^3$$~~

$$R_{\lambda\beta} = \frac{\partial \Gamma_{\beta\lambda}^{\sigma}}{\partial x^{\sigma}} - \frac{\partial \Gamma_{\sigma\lambda}^{\sigma}}{\partial x^{\beta}} + \Gamma_{\sigma\lambda}^{\sigma} \Gamma_{\beta\sigma}^{\nu} - \Gamma_{\beta\lambda}^{\sigma} \Gamma_{\sigma\sigma}^{\nu}$$

$$R_{00}, R_{01}, R_{11}, R_{22}, R_{33} \neq 0$$

$$1) R_{00} = \frac{\partial \Gamma_{00}^{\sigma}}{\partial x^{\sigma}} - \frac{\partial \Gamma_{\sigma 0}^{\sigma}}{\partial x^0} + \Gamma_{\sigma\lambda}^{\sigma} \Gamma_{00}^{\lambda} - \Gamma_{0\lambda}^{\sigma} \Gamma_{\sigma 0}^{\lambda} =$$

$$= \frac{\partial \Gamma_{00}^0}{\partial x^0} + \frac{\partial \Gamma_{00}^1}{\partial x^1} - \frac{\partial \Gamma_{00}^0}{\partial x^0} - \frac{\partial \Gamma_{10}^1}{\partial x^0} - \frac{\partial \Gamma_{20}^2}{\partial x^0} - \frac{\partial \Gamma_{30}^3}{\partial x^0} +$$

$$\Gamma_{20}^1 = \Gamma_{30}^3$$

$$+ \Gamma_{\sigma 0}^{\sigma} \Gamma_{00}^0 + \Gamma_{\sigma 1}^{\sigma} \Gamma_{00}^1 + \Gamma_{\sigma 2}^{\sigma} \Gamma_{00}^2 + \Gamma_{\sigma 3}^{\sigma} \Gamma_{00}^3 -$$

$$- \Gamma_{00}^{\sigma} \Gamma_{\sigma 0}^0 - \Gamma_{01}^{\sigma} \Gamma_{\sigma 0}^1 - \Gamma_{02}^{\sigma} \Gamma_{\sigma 0}^2 - \Gamma_{03}^{\sigma} \Gamma_{\sigma 0}^3 =$$

$$= \frac{\partial \Gamma_{00}^1}{\partial x} - \frac{\partial \Gamma_{10}^1}{\partial t} - 2 \frac{\partial \Gamma_{20}^2}{\partial t} +$$

$$+ \Gamma_{00}^0 \Gamma_{00}^0 + \Gamma_{10}^1 \Gamma_{00}^0 + \Gamma_{10}^2 \Gamma_{00}^0 + \Gamma_{30}^3 \Gamma_{00}^0 +$$

$$+ \Gamma_{01}^0 \Gamma_{00}^1 + \Gamma_{11}^1 \Gamma_{00}^1 + \Gamma_{21}^2 \Gamma_{00}^1 + \Gamma_{31}^3 \Gamma_{00}^1 -$$

~~$$+ \Gamma_{02}^0 \Gamma_{00}^2 + \Gamma_{12}^1 \Gamma_{00}^2 + \Gamma_{22}^2 \Gamma_{00}^2 + \Gamma_{32}^3 \Gamma_{00}^2 +$$~~

~~$$+ \Gamma_{03}^0 \Gamma_{00}^3$$~~

~~$$- \Gamma_{00}^0 \Gamma_{00}^0 - \Gamma_{00}^1 \Gamma_{10}^0 - \Gamma_{00}^2 \Gamma_{20}^0 - \Gamma_{00}^3 \Gamma_{30}^0 -$$~~

~~$$- \Gamma_{01}^0 \Gamma_{00}^1 - \Gamma_{01}^1 \Gamma_{10}^1 - \Gamma_{01}^2 \Gamma_{20}^1 - \Gamma_{01}^3 \Gamma_{30}^1 -$$~~

~~$$- \Gamma_{02}^0 \Gamma_{00}^2 - \Gamma_{02}^1 \Gamma_{10}^2 - \Gamma_{02}^2 \Gamma_{20}^2 - \Gamma_{03}^3 \Gamma_{30}^2 -$$~~

~~$$- \Gamma_{03}^0 \Gamma_{00}^3 - \Gamma_{03}^1 \Gamma_{10}^3 - \Gamma_{03}^2 \Gamma_{20}^3 - \Gamma_{03}^3 \Gamma_{30}^3 =$$~~

$$= \frac{\partial \Gamma_{00}^1}{\partial x} - \frac{\partial \Gamma_{10}^1}{\partial t} - 2 \frac{\partial \Gamma_{20}^2}{\partial t} +$$

$$+ \Gamma_{01}^1 \Gamma_{00}^0 + 2 \Gamma_{02}^2 \Gamma_{00}^0 + \Gamma_{11}^1 \Gamma_{00}^1 + 2 \Gamma_{12}^2 \Gamma_{00}^1 -$$

$$- \Gamma_{01}^0 \Gamma_{00}^1 - \Gamma_{01}^1 \Gamma_{01}^1 - 2 \Gamma_{02}^2 \Gamma_{01}^2 = R_{00}$$

$$2) R_{01} = \frac{\partial \Gamma_{10}^0}{\partial x^0} - \frac{\partial \Gamma_{00}^0}{\partial x^1} +$$

$$+ \Gamma_{01}^0 \Gamma_{10}^1 - \Gamma_{11}^0 \Gamma_{00}^1 =$$

$$= \frac{\partial \Gamma_{01}^0}{\partial t} + \frac{\partial \Gamma_{01}^1}{\partial x} - \frac{\partial \Gamma_{00}^0}{\partial x} - \frac{\partial \Gamma_{01}^1}{\partial x} - \frac{\partial \Gamma_{02}^2}{\partial x} - \frac{\partial \Gamma_{03}^3}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{01}^0 + \Gamma_{01}^0 \Gamma_{01}^1 + \Gamma_{02}^2 \Gamma_{01}^2 + \Gamma_{03}^3 \Gamma_{01}^3 -$$

$$- \Gamma_{10}^0 \Gamma_{00}^0 - \Gamma_{11}^0 \Gamma_{00}^1 - \Gamma_{12}^0 \Gamma_{00}^2 - \Gamma_{13}^0 \Gamma_{00}^3 =$$

$$= \frac{\partial \Gamma_{01}^0}{\partial t} + \frac{\partial \Gamma_{10}^0}{\partial x} + \Gamma_{10}^0 \Gamma_{01}^0 + \Gamma_{20}^2 \Gamma_{01}^2 + \Gamma_{30}^3 \Gamma_{01}^3 +$$

$$+ \Gamma_{01}^0 \Gamma_{01}^1 + \Gamma_{11}^1 \Gamma_{01}^1 + \Gamma_{21}^2 \Gamma_{01}^2 + \Gamma_{31}^3 \Gamma_{01}^3 -$$

$$- \frac{\partial \Gamma_{10}^0}{\partial x} - \frac{\partial \Gamma_{10}^1}{\partial x} - \frac{\partial \Gamma_{10}^2}{\partial x} - \frac{\partial \Gamma_{10}^3}{\partial x} -$$

$$- \Gamma_{11}^0 \Gamma_{00}^1 - \Gamma_{11}^1 \Gamma_{10}^1 - \Gamma_{11}^2 \Gamma_{10}^2 - \Gamma_{11}^3 \Gamma_{10}^3 -$$

$$- \Gamma_{12}^0 \Gamma_{00}^2 - \Gamma_{12}^1 \Gamma_{10}^2 - \Gamma_{12}^2 \Gamma_{10}^2 - \Gamma_{12}^3 \Gamma_{10}^3 -$$

$$- \Gamma_{13}^0 \Gamma_{00}^3 - \Gamma_{13}^1 \Gamma_{10}^3 - \Gamma_{13}^2 \Gamma_{10}^3 - \Gamma_{13}^3 \Gamma_{10}^3 =$$

$$= \frac{\partial \Gamma_{01}^0}{\partial t} - \frac{\partial \Gamma_{00}^0}{\partial x} - 2 \frac{\partial \Gamma_{02}^2}{\partial x} +$$

$$+ 2 \Gamma_{02}^2 \Gamma_{01}^0 + \Gamma_{01}^0 \Gamma_{01}^1 + 2 \Gamma_{12}^2 \Gamma_{01}^1 -$$

$$- \Gamma_{11}^0 \Gamma_{00}^1 - 2 \Gamma_{12}^2 \Gamma_{01}^2 = R_{01}$$

$$3) R_{11} = \frac{\partial \Gamma_{11}^0}{\partial x^0} - \frac{\partial \Gamma_{01}^0}{\partial x^1} + \Gamma_{00}^0 \Gamma_{11}^0 - \Gamma_{10}^0 \Gamma_{01}^0 =$$

$$= \frac{\partial \Gamma_{11}^0}{\partial x^0} + \frac{\partial \Gamma_{11}^1}{\partial x^1} - \frac{\partial \Gamma_{01}^0}{\partial x^1} - \frac{\partial \Gamma_{11}^1}{\partial x^1} - \frac{\partial \Gamma_{21}^2}{\partial x^1} - \frac{\partial \Gamma_{31}^3}{\partial x^1} +$$

$$+ \Gamma_{00}^0 \Gamma_{11}^0 + \Gamma_{01}^0 \Gamma_{11}^1 + \Gamma_{02}^0 \Gamma_{11}^2 + \Gamma_{03}^0 \Gamma_{11}^3 -$$

$$- \Gamma_{10}^0 \Gamma_{01}^0 - \Gamma_{11}^0 \Gamma_{01}^1 - \Gamma_{12}^0 \Gamma_{01}^2 - \Gamma_{13}^0 \Gamma_{01}^3 =$$

$$= \frac{\partial \Gamma_{11}^0}{\partial t} - \frac{\partial \Gamma_{01}^0}{\partial x} - 2 \frac{\partial \Gamma_{12}^2}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{11}^0 + \Gamma_{10}^1 \Gamma_{11}^0 + \Gamma_{20}^2 \Gamma_{11}^0 + \Gamma_{30}^3 \Gamma_{11}^0 +$$

$$+ \Gamma_{01}^0 \Gamma_{11}^1 + \Gamma_{11}^1 \Gamma_{11}^1 + \Gamma_{21}^2 \Gamma_{11}^1 + \Gamma_{31}^3 \Gamma_{11}^1 -$$

$$- \Gamma_{10}^0 \Gamma_{01}^0 - \Gamma_{10}^1 \Gamma_{11}^0 - \Gamma_{20}^2 \Gamma_{11}^0 - \Gamma_{30}^3 \Gamma_{11}^0 -$$

$$- \Gamma_{11}^0 \Gamma_{01}^1 - \Gamma_{11}^1 \Gamma_{11}^1 - \Gamma_{21}^2 \Gamma_{11}^1 - \Gamma_{31}^3 \Gamma_{11}^1 -$$

$$- \Gamma_{12}^0 \Gamma_{01}^2 - \Gamma_{12}^1 \Gamma_{11}^2 - \Gamma_{22}^2 \Gamma_{11}^2 - \Gamma_{32}^3 \Gamma_{11}^2 -$$

$$- \Gamma_{13}^0 \Gamma_{01}^3 - \Gamma_{13}^1 \Gamma_{11}^3 - \Gamma_{23}^2 \Gamma_{11}^3 - \Gamma_{33}^3 \Gamma_{11}^3 =$$

$$= \frac{\partial \Gamma_{11}^0}{\partial t} - \frac{\partial \Gamma_{01}^0}{\partial x} - 2 \frac{\partial \Gamma_{12}^2}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{11}^0 + 2 \Gamma_{02}^2 \Gamma_{11}^0 + \Gamma_{04}^4 \Gamma_{11}^4 + 2 \Gamma_{12}^2 \Gamma_{11}^2 -$$

$$- \Gamma_{01}^0 \Gamma_{01}^0 - \Gamma_{11}^0 \Gamma_{01}^1 - 2 \Gamma_{12}^2 \Gamma_{11}^2 = R_{11}$$

$$4) R_{22} = \frac{\partial \Gamma_{22}^0}{\partial x^0} + \Gamma_{02}^0 \Gamma_{22}^0 - \Gamma_{22}^0 \Gamma_{02}^0 =$$

$$= \frac{\partial \Gamma_{22}^0}{\partial t} + \frac{\partial \Gamma_{22}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{22}^0 + \Gamma_{04}^0 \Gamma_{22}^1 + \cancel{\Gamma_{02}^0 \Gamma_{22}^2} + \cancel{\Gamma_{03}^0 \Gamma_{22}^3} -$$

$$- \Gamma_{20}^0 \Gamma_{02}^0 - \Gamma_{22}^0 \Gamma_{02}^1 - \Gamma_{22}^0 \Gamma_{02}^2 - \cancel{\Gamma_{23}^0 \Gamma_{02}^3} =$$

$$= \frac{\partial \Gamma_{22}^0}{\partial t} + \frac{\partial \Gamma_{22}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{22}^0 + \Gamma_{10}^1 \Gamma_{22}^0 + \cancel{\Gamma_{20}^2 \Gamma_{22}^0} + \Gamma_{30}^3 \Gamma_{22}^0 +$$

$$+ \Gamma_{01}^0 \Gamma_{22}^1 + \Gamma_{11}^1 \Gamma_{22}^1 + \cancel{\Gamma_{21}^2 \Gamma_{22}^1} + \Gamma_{31}^3 \Gamma_{22}^1 -$$

$$- \Gamma_{20}^0 \Gamma_{02}^0 - \cancel{\Gamma_{20}^1 \Gamma_{02}^1} - \cancel{\Gamma_{20}^2 \Gamma_{02}^2} - \cancel{\Gamma_{20}^3 \Gamma_{02}^3} -$$

$$- \cancel{\Gamma_{21}^0 \Gamma_{02}^1} - \cancel{\Gamma_{21}^1 \Gamma_{02}^1} - \cancel{\Gamma_{21}^2 \Gamma_{02}^1} - \cancel{\Gamma_{21}^3 \Gamma_{02}^1} -$$

$$- \Gamma_{22}^0 \Gamma_{02}^2 - \Gamma_{22}^1 \Gamma_{02}^2 - \cancel{\Gamma_{22}^2 \Gamma_{02}^2} - \cancel{\Gamma_{22}^3 \Gamma_{02}^2} =$$

$$= \frac{\partial \Gamma_{22}^0}{\partial t} + \frac{\partial \Gamma_{22}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{22}^0 + \cancel{\Gamma_{02}^1 \Gamma_{22}^0} + \Gamma_{03}^3 \Gamma_{22}^0 + \cancel{\Gamma_{02}^0 \Gamma_{22}^1} +$$

$$+ \Gamma_{11}^1 \Gamma_{22}^1 + \Gamma_{23}^3 \Gamma_{22}^1 -$$

$$- \cancel{\Gamma_{02}^0 \Gamma_{02}^0} - \Gamma_{22}^0 \Gamma_{02}^2 - \Gamma_{22}^1 \Gamma_{02}^2 = R_{22}$$

$$5) R_{33} = \frac{\partial \Gamma_{33}^0}{\partial x^0} + \Gamma_{00}^0 \Gamma_{33}^0 - \Gamma_{30}^0 \Gamma_{03}^0 =$$

$$= \frac{\partial \Gamma_{33}^0}{\partial t} + \frac{\partial \Gamma_{33}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{33}^0 + \Gamma_{01}^0 \Gamma_{33}^1 + \Gamma_{02}^0 \Gamma_{33}^2 + \Gamma_{03}^0 \Gamma_{33}^3 -$$

$$- \Gamma_{30}^0 \Gamma_{03}^0 - \Gamma_{31}^0 \Gamma_{03}^1 - \Gamma_{32}^0 \Gamma_{03}^2 - \Gamma_{33}^0 \Gamma_{03}^3 =$$

$$= \frac{\partial \Gamma_{33}^0}{\partial t} + \frac{\partial \Gamma_{33}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{23}^0 + \Gamma_{01}^1 \Gamma_{33}^0 + \Gamma_{02}^2 \Gamma_{33}^0 + \Gamma_{03}^3 \Gamma_{33}^0 +$$

$$+ \Gamma_{04}^0 \Gamma_{33}^1 + \Gamma_{11}^1 \Gamma_{33}^1 + \Gamma_{21}^2 \Gamma_{33}^1 + \Gamma_{31}^3 \Gamma_{33}^1 -$$

$$- \Gamma_{30}^0 \Gamma_{03}^0 - \Gamma_{30}^1 \Gamma_{23}^0 - \Gamma_{30}^2 \Gamma_{23}^0 - \Gamma_{30}^3 \Gamma_{23}^0 -$$

$$- \Gamma_{31}^0 \Gamma_{03}^1 - \Gamma_{31}^1 \Gamma_{23}^1 - \Gamma_{31}^2 \Gamma_{23}^1 - \Gamma_{31}^3 \Gamma_{23}^1 -$$

$$- \Gamma_{33}^0 \Gamma_{03}^3 - \Gamma_{33}^1 \Gamma_{23}^3 - \Gamma_{33}^2 \Gamma_{23}^3 - \Gamma_{33}^3 \Gamma_{23}^3 =$$

$$= \frac{\partial \Gamma_{33}^0}{\partial t} + \frac{\partial \Gamma_{33}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{33}^0 + \Gamma_{01}^1 \Gamma_{33}^0 + \Gamma_{02}^2 \Gamma_{33}^0 + \Gamma_{04}^0 \Gamma_{33}^1 +$$

$$+ \Gamma_{11}^1 \Gamma_{33}^1 + \Gamma_{21}^2 \Gamma_{33}^1 - \Gamma_{13}^3 \Gamma_{33}^1 - \Gamma_{33}^0 \Gamma_{03}^3 = R_{33}$$

$$\dot{\Delta}^1 = \frac{1}{2\delta - \beta^2}$$

(1)

$$\begin{aligned}
 R_{11} &= \left\{ \frac{1}{\Delta} \left[\gamma \beta' - \frac{1}{2} \gamma \ddot{\gamma} - \frac{1}{2} \beta \gamma'' \right] \right\}' - \\
 &\quad - \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \alpha' - \frac{1}{2} \beta \ddot{\gamma} \right] \right\}' - \left\{ \frac{\omega'}{\omega} \right\}' + \\
 &\quad + \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \ddot{\alpha} - \beta \dot{\beta} + \frac{1}{2} \beta \alpha' \right] \right\} \left\{ \frac{1}{\Delta} \left[\gamma \beta' - \frac{1}{2} \gamma \ddot{\gamma} - \frac{1}{2} \beta \gamma'' \right] \right\} + \\
 &\quad + \left\{ \frac{\dot{\omega}}{\omega} \right\} \left\{ \frac{1}{\Delta} \left[\gamma \beta' - \frac{1}{2} \gamma \ddot{\gamma} - \frac{1}{2} \beta \gamma'' \right] \right\} + \\
 &\quad + \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \alpha' - \frac{1}{2} \beta \ddot{\gamma} \right] \right\} \left\{ \frac{1}{\Delta} \left[-\beta \beta' - \frac{1}{2} \alpha \gamma' + \frac{1}{2} \beta \ddot{\gamma} \right] \right\} + \\
 &\quad + \left\{ \frac{\omega'}{\omega} \right\} \left\{ \frac{1}{\Delta} \left[-\beta \beta' - \frac{1}{2} \alpha \gamma' + \frac{1}{2} \beta \ddot{\gamma} \right] \right\} - \\
 &\quad - \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \alpha' - \frac{1}{2} \beta \ddot{\gamma} \right] \right\} \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \alpha' - \frac{1}{2} \beta \ddot{\gamma} \right] \right\} - \\
 &\quad - \left\{ \frac{1}{\Delta} \left[\gamma \beta' - \frac{1}{2} \gamma \ddot{\gamma} - \frac{1}{2} \beta \gamma'' \right] \right\} \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \beta \alpha' + \frac{1}{2} \alpha \ddot{\gamma} \right] \right\} - \\
 &\quad - \frac{1}{2} \left\{ \frac{\omega'}{\omega} \right\} \left\{ \frac{\omega'}{\omega} \right\}
 \end{aligned}$$

$$\begin{aligned}
 R_{22} &= \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \gamma \dot{\omega} + \frac{1}{2} \beta \omega' \right] \right\}' + \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \beta \dot{\omega} - \frac{1}{2} \alpha \omega' \right] \right\}' + \quad (2) \\
 &+ \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \dot{\alpha} - \beta \dot{\beta} + \frac{1}{2} \beta \alpha' \right] \right\} \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \gamma \dot{\omega} + \frac{1}{2} \beta \omega' \right] \right\} + \\
 &+ \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \beta \alpha' + \frac{1}{2} \alpha \dot{\gamma} \right] \right\} \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \gamma \dot{\omega} + \frac{1}{2} \beta \omega' \right] \right\} + \\
 &+ \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \alpha' - \frac{1}{2} \beta \dot{\gamma} \right] \right\} \left\{ \left[\frac{1}{2} \beta \dot{\omega} - \frac{1}{2} \alpha \omega' \right] \frac{1}{\Delta} \right\} + \\
 &+ \left\{ \frac{1}{\Delta} \left[-\beta \beta' - \frac{1}{2} \alpha \gamma' + \frac{1}{2} \beta \dot{\gamma} \right] \right\} \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \beta \dot{\omega} - \frac{1}{2} \alpha \omega' \right] \right\}
 \end{aligned}$$

$$R_0^0 = g^{00} R_{00} + g^{01} R_{10}$$

$$R_0^1 = g^{01} R_{00} + g^{11} R_{01}$$

$$R_1^1 = g^{01} R_{01} + g^{11} R_{11}$$

$$R = R_0^0 + R_1^1 + 2R_2^2$$

$$R_2^2 = R_3^3 = g^{22} R_{22}$$

$$\begin{aligned}
 R_{01} &= \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \alpha' - \frac{1}{2} \beta \dot{\gamma} \right] \right\}' - \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \dot{\alpha} - \beta \dot{\beta} + \frac{1}{2} \beta \alpha' \right] \right\}' - \left\{ \frac{\dot{\omega}}{\omega} \right\}' + \\
 &+ \left\{ \frac{\dot{\omega}}{\omega} \right\} \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \alpha' - \frac{1}{2} \beta \dot{\gamma} \right] \right\} + \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \alpha' - \frac{1}{2} \beta \dot{\gamma} \right] \right\} \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \beta \alpha' + \frac{1}{2} \alpha \dot{\gamma} \right] \right\} + \\
 &+ \left\{ \frac{\omega'}{\omega} \right\} \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \beta \alpha' + \frac{1}{2} \alpha \dot{\gamma} \right] \right\} - \\
 &- \left\{ \frac{1}{\Delta} \left[\gamma \beta' - \frac{1}{2} \gamma \dot{\gamma} - \frac{1}{2} \beta \alpha' \right] \right\} \left\{ \frac{1}{\Delta} \left[\alpha \dot{\beta} - \frac{1}{2} \alpha \alpha' - \frac{1}{2} \beta \alpha' \right] \right\} - \\
 &- \frac{1}{2} \left\{ \frac{\omega'}{\omega} \right\} \left\{ \frac{\dot{\omega}}{\omega} \right\}
 \end{aligned}$$

$$\begin{aligned}
R_{00} &= \left\{ \frac{1}{\Delta} \left[\alpha \dot{\beta} - \frac{1}{2} \alpha \alpha' - \frac{1}{2} \beta \dot{\alpha} \right] \right\}' - \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \beta \alpha' + \frac{1}{2} \alpha \dot{\gamma} \right] \right\}' - \\
&\quad - \left\{ \frac{\dot{\omega}}{\omega} \right\}' + \\
&\quad + \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \beta \alpha' + \frac{1}{2} \alpha \dot{\gamma} \right] \right\} \left\{ \frac{1}{\Delta} \left[\gamma \dot{\alpha} - \beta \dot{\beta} + \frac{1}{2} \beta \alpha' \right] \right\} + \\
&\quad + \left\{ \frac{\dot{\omega}}{\omega} \right\} \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \dot{\alpha} - \beta \dot{\beta} + \frac{1}{2} \beta \alpha' \right] \right\} + \\
&\quad + \left\{ \frac{1}{\Delta} \left[-\beta \beta' - \frac{1}{2} \alpha \gamma' + \frac{1}{2} \beta \dot{\gamma} \right] \right\} \left\{ \frac{1}{\Delta} \left[\alpha \dot{\beta} - \frac{1}{2} \alpha \alpha' - \frac{1}{2} \beta \dot{\alpha} \right] \right\} + \\
&\quad + \left\{ \frac{\omega'}{\omega} \right\} \left\{ \frac{1}{\Delta} \left[\alpha \dot{\beta} - \frac{1}{2} \alpha \alpha' - \frac{1}{2} \beta \dot{\alpha} \right] \right\} - \\
&\quad - \left\{ \frac{1}{\Delta} \left[\frac{1}{2} \gamma \alpha' - \frac{1}{2} \beta \dot{\gamma} \right] \right\} \left\{ \frac{1}{\Delta} \left[\alpha \dot{\beta} - \frac{1}{2} \alpha \alpha' - \frac{1}{2} \beta \dot{\alpha} \right] \right\} - \\
&\quad - \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \beta \alpha' + \frac{1}{2} \alpha \dot{\gamma} \right] \right\} \left\{ \frac{1}{\Delta} \left[-\frac{1}{2} \beta \alpha' + \frac{1}{2} \alpha \dot{\gamma} \right] \right\} - \\
&\quad - \frac{1}{2} \left\{ \frac{\dot{\omega}}{\omega} \right\} \left\{ \frac{\dot{\omega}}{\omega} \right\}
\end{aligned}$$

$$1) R_{00} = \frac{\partial \Gamma_{00}^1}{\partial x} - \frac{\partial \Gamma_{01}^1}{\partial t} - 2 \frac{\partial \Gamma_{02}^2}{\partial t} +$$

$$+ \Gamma_{01}^1 \Gamma_{00}^0 + 2 \Gamma_{02}^2 \Gamma_{00}^0 + \Gamma_{11}^1 \Gamma_{00}^1 + 2 \Gamma_{12}^2 \Gamma_{00}^1 -$$

$$- \Gamma_{01}^0 \Gamma_{00}^1 - \Gamma_{01}^1 \Gamma_{01}^1 - 2 \Gamma_{02}^2 \Gamma_{02}^2$$

$$2) R_{01} = \frac{\partial \Gamma_{01}^0}{\partial t} - \frac{\partial \Gamma_{00}^0}{\partial x} - 2 \frac{\partial \Gamma_{02}^2}{\partial x} +$$

$$+ 2 \Gamma_{02}^2 \Gamma_{01}^0 + \Gamma_{01}^0 \Gamma_{01}^1 + 2 \Gamma_{12}^2 \Gamma_{01}^1 - \Gamma_{11}^0 \Gamma_{00}^1 - 2 \Gamma_{12}^2 \Gamma_{02}^2$$

$$3) R_{11} = \frac{\partial \Gamma_{11}^0}{\partial t} - \frac{\partial \Gamma_{01}^0}{\partial x} - 2 \frac{\partial \Gamma_{12}^2}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{11}^0 + 2 \Gamma_{02}^2 \Gamma_{11}^0 + \Gamma_{01}^0 \Gamma_{11}^1 + 2 \Gamma_{12}^2 \Gamma_{11}^1 -$$

$$- \Gamma_{01}^0 \Gamma_{01}^0 - \Gamma_{11}^0 \Gamma_{01}^1 - 2 \Gamma_{12}^2 \Gamma_{12}^2$$

$$4) R_{22} = \frac{\partial \Gamma_{22}^0}{\partial t} + \frac{\partial \Gamma_{22}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{22}^0 + \Gamma_{01}^1 \Gamma_{22}^0 + \cancel{\Gamma_{03}^3 \Gamma_{22}^0} + \Gamma_{01}^0 \Gamma_{22}^1 +$$

$$+ \Gamma_{11}^1 \Gamma_{22}^1 + \cancel{\Gamma_{13}^3 \Gamma_{22}^1} - \cancel{\Gamma_{02}^2 \Gamma_{02}^0} - \cancel{\Gamma_{22}^0 \Gamma_{02}^2} - \cancel{\Gamma_{22}^1 \Gamma_{12}^2}$$

$$5) R_{33} = \frac{\partial \Gamma_{33}^0}{\partial t} + \frac{\partial \Gamma_{33}^1}{\partial x} +$$

$$+ \Gamma_{00}^0 \Gamma_{33}^0 + \Gamma_{01}^1 \Gamma_{33}^0 + \Gamma_{02}^2 \Gamma_{33}^0 + \Gamma_{01}^0 \Gamma_{33}^1 +$$

$$+ \Gamma_{11}^1 \Gamma_{33}^1 + \Gamma_{12}^2 \Gamma_{33}^1 - \Gamma_{33}^0 \Gamma_{03}^3 - \Gamma_{33}^1 \Gamma_{13}^3$$

6) $R_{\dots} = \dots$

$$1) R_{00} = \frac{1}{2} \left(\frac{1}{\Delta} \right)^2 \left\{ \alpha \alpha' \left[\beta \beta' + \frac{1}{2} \alpha \gamma' - \frac{1}{2} \gamma \alpha' - \frac{\omega'}{\omega} \right] \right\} - \frac{1}{2} \left(\frac{1}{\Delta} \right)^2 (\beta \alpha')^2 -$$

$$- \frac{1}{2} \left\{ \frac{1}{\Delta} \alpha \alpha' \right\}'.$$

$$2) R_{01} = \frac{1}{2} \left(\frac{1}{\Delta} \right)^2 \left\{ \alpha \alpha' \left[\gamma \beta' - \frac{1}{2} \beta \gamma' \right] \right\} - \frac{1}{2} \left(\frac{1}{\Delta} \right)^2 \left\{ \beta \alpha' \left[\frac{1}{2} \gamma \alpha' - \frac{\omega'}{\Delta \omega} \right] \right\} -$$

$$- \frac{1}{2} \left\{ \frac{1}{\Delta} \beta \alpha' \right\}'.$$

$$3) R_{11} = \left(\frac{1}{\Delta} \right)^2 \left\{ \beta \alpha' \left[\gamma \beta' - \frac{1}{2} \beta \gamma' \right] \right\} - \frac{1}{\Delta} \left\{ \frac{\omega'}{\omega} \right\} \left\{ \beta \beta' + \frac{1}{2} \alpha \gamma' \right\} -$$

$$- \frac{1}{2} \left(\frac{\omega'}{\omega} \right)^2 - \frac{1}{2} \left(\frac{1}{\Delta} \right)^2 \left\{ \gamma \alpha' \left[\frac{1}{2} \gamma \alpha' + \beta \beta' + \frac{1}{2} \alpha \gamma' \right] \right\} -$$

$$- \left\{ \frac{\omega'}{\omega} \right\}' - \frac{1}{2} \left\{ \frac{1}{\Delta} \gamma \alpha' \right\}'.$$

$$4) R_{22} = R_{33} = \frac{1}{2} \left(\frac{1}{\Delta} \right)^2 \left\{ \alpha \omega' \left[\beta \beta' - \frac{1}{2} \gamma \alpha' + \frac{1}{2} \alpha \gamma' \right] \right\} -$$

$$- \frac{1}{2} \left\{ \frac{1}{\Delta} \alpha \omega' \right\}'.$$

Агномерний стационарний випадок
 $\left(\frac{\partial}{\partial x^0} = 0 \right)$.

$$g_{xy} = \begin{pmatrix} \alpha & 0 \\ 0 & \gamma \end{pmatrix}$$

$$g^{xy} = \begin{pmatrix} 1/\alpha & 0 \\ 0 & 1/\gamma \end{pmatrix}$$

$$\begin{aligned} ({}^1_1) &= \frac{1}{2} \cdot \frac{\alpha'}{\alpha} \\ ({}^1_2) &= \frac{1}{2} \cdot \frac{\alpha'}{\alpha} \\ ({}^2_2) &= -\frac{1}{2} \cdot \frac{\gamma'}{\gamma} \\ ({}^2_{11}) &= -\frac{1}{2} \cdot \frac{\alpha'}{\gamma} \\ ({}^2_{12}) &= \frac{1}{2} \cdot \frac{\alpha'}{\gamma} \\ ({}^2_{22}) &= \frac{1}{2} \cdot \frac{\gamma'}{\gamma} \end{aligned}$$

$$R'_{212} = \frac{\partial ({}^1_{22})}{\partial x} - \frac{\partial ({}^1_{12})}{\partial y} + ({}^1_{11})({}^1_{22}) + ({}^1_{12})({}^2_{22}) - ({}^1_{21})({}^1_{12}) - ({}^1_{22})({}^2_{12})$$

$$R_{1212} = -\frac{1}{2} \frac{\gamma''}{\gamma} - \frac{1}{2} \frac{\alpha''}{\alpha} + \frac{1}{4} \frac{\alpha'^2}{\alpha^2} + \frac{1}{4} \frac{\alpha'^2}{\alpha^2} + \frac{1}{4} \frac{\gamma'^2}{\gamma^2} + \frac{1}{4} \frac{\alpha'^2}{\gamma^2}$$

$$\begin{aligned} R_{11} &= \frac{1}{\gamma} R_{1212} \\ R_{22} &= \frac{1}{\alpha} R_{1212} \\ R_{12} &= 0 \\ R &= \frac{2}{\alpha\gamma} R_{1212} \end{aligned}$$

Уравнения Римана:
 $R_{1212} = 0.$

$$\left. \begin{aligned} -\ddot{\gamma} + \frac{1}{2} \frac{\alpha'' \dot{\gamma}}{\alpha} + \frac{1}{2} \frac{\dot{\gamma}^2}{\gamma} &= 0 \\ -\frac{\ddot{\gamma}}{\gamma} + \frac{1}{2} \frac{\alpha''}{\alpha} + \frac{1}{2} \frac{\dot{\gamma}}{\gamma} &= 0 \end{aligned} \right\} \text{В случае, когда функции } \alpha, \gamma \text{ зависят только от одной координаты}$$

$\frac{\alpha\gamma}{\dot{\gamma}^2} = \text{Const} \leftarrow \text{Это также константа.}$

$$\frac{\dot{\gamma}^2}{\gamma} = C \cdot \alpha$$

~~$\alpha = \gamma \Rightarrow \ln \gamma = Cx$~~
 ~~$\gamma = e^{Cx}$~~

$$\alpha = x \Rightarrow \dot{\gamma}^2 = Cx\gamma$$

$$2\sqrt{\gamma} = \frac{2}{3} C_1 \cdot x^{3/2} + C_2$$

$$-\frac{\ddot{y}}{y} + \frac{1}{2} \frac{\dot{a}}{a} + \frac{1}{2} \frac{\dot{y}}{y} = 0.$$

$$\left(-\ln y + \frac{1}{2} \ln a + \frac{1}{2} \ln y\right)' = 0.$$

$$\ln \frac{\sqrt{ay}}{y^{\frac{1}{2}}} = \text{const.}$$

$$\frac{\sqrt{ay}}{y^{\frac{1}{2}}} = \text{const.}$$

$$-\frac{1}{2}\ddot{\gamma} - \frac{1}{2}\alpha'' + \frac{1}{4}\frac{\dot{\gamma}}{\alpha} + \frac{1}{4}\frac{\alpha'^2}{\alpha} + \frac{1}{4}\frac{\dot{\gamma}^2}{\gamma} + \frac{1}{4}\frac{\alpha'\gamma'}{\gamma} = 0 \quad (1)$$

Предположим, что $\alpha = f(x) \cdot g(t)$ и $\gamma = f^*(x) \cdot g^*(t)$,

Тогда это уравнение разбивается на два уравнения:

$$\begin{cases} -\frac{1}{2}\ddot{\gamma} + \frac{1}{4}\frac{\dot{\gamma}}{\alpha} + \frac{1}{4}\frac{\dot{\gamma}^2}{\gamma} = 0 & (1) \end{cases}$$

$$\begin{cases} -\frac{1}{2}\alpha'' + \frac{1}{4}\frac{\alpha'\gamma'}{\gamma} + \frac{1}{4}\frac{\alpha'^2}{\alpha} = 0 & (2) \end{cases}$$

которые удовлетворяются одновременно, при этом

$$\begin{aligned} f(x) &\neq 0, \quad g(t) \neq 0 \\ f^*(x) &\neq 0, \quad g^*(t) \neq 0. \end{aligned}$$

В ур-х (1), (2) ср-ная α и γ удовлетворяют только всей временной координатной в (1) и только x -и в (2).

Оба уравнения сводятся к тому же соотношению

$$\begin{cases} \frac{\alpha\gamma}{\dot{\gamma}^2} = \text{const} \\ \frac{\alpha\gamma}{\alpha'^2} = \text{const}^* \end{cases}$$

по следующей схеме:

$$-\frac{\ddot{\gamma}}{\dot{\gamma}} + \frac{1}{2}\frac{\dot{\alpha}}{\alpha} + \frac{1}{2}\frac{\dot{\gamma}}{\gamma} = 0 \quad (\dot{\gamma} \neq 0)$$

$$\left(-\ln \dot{\gamma} + \frac{1}{2}\ln \alpha + \frac{1}{2}\ln \gamma\right)' = 0$$

$$\ln \frac{\sqrt{\alpha\gamma}}{\dot{\gamma}} = \text{const} \Rightarrow \frac{\sqrt{\alpha\gamma}}{\dot{\gamma}} = \text{const}$$

Решим уравнение $\frac{d^2 x}{dt^2} = \text{const}$.

(2)

Сделаем предположение относительно x . Поскольку считаем, что в пространстве распр-я плоская симметрия, то возьмем

$$x = A e^{i\omega t}$$

$$\frac{A e^{i\omega t}}{j^2} = C$$

Это уравнение лучше всего решать на комплексной плоскости.

Решим это уравнение ищем x в виде:

$$x = B e^{-i\Omega t}$$

$$\dot{x} = B(-i\Omega) e^{-i\Omega t}, \Rightarrow$$

$$-\frac{A e^{i\omega t}}{B \Omega^2 e^{-i\Omega t}} = C$$

$$e^{i(\omega+\Omega)t} = -\frac{BC}{A} \Omega^2$$

$$\alpha = F(t)\phi(x)$$

$$\gamma = f(t)\psi(x)$$

$$\left(-\frac{1}{2} \ddot{f} \psi + \frac{1}{4} \dot{F} \dot{f} \psi + \frac{1}{4} \ddot{f} \frac{f}{f} \psi \right) + \left(-\frac{1}{2} F \phi'' + \frac{1}{4} \phi'^2 + \frac{1}{4} F \frac{F \phi' \psi'}{\psi} \right) = 0.$$

$\frac{1}{4} \psi \dot{f}$

$$\left(\cancel{\frac{1}{4} \psi \dot{f}} - \frac{1}{2} \ddot{f} \frac{f}{f} + \frac{1}{4} \dot{F} \frac{f}{f} + \frac{1}{4} \ddot{f} \frac{f}{f} \right) + \dots$$

$\frac{1}{4} \psi \dot{f}$

$$\left(\cancel{\frac{1}{4} \psi \dot{f}} - \frac{1}{2} \frac{\phi''}{\phi} + \frac{1}{4} \frac{\phi'^2}{\phi^2} + \frac{1}{4} \frac{\phi' \psi'}{\psi} \right) = 0.$$

$\psi \cdot \psi \propto x$

$$\left(-\frac{1}{2} \ddot{f} \frac{f}{f} + \frac{1}{4} \dot{F} \frac{f}{f} + \frac{1}{4} \ddot{f} \frac{f}{f} \right) = 0$$

$$\left(-\frac{1}{2} \frac{\phi''}{\phi} + \frac{1}{4} \frac{\phi'^2}{\phi^2} + \frac{1}{4} \frac{\phi' \psi'}{\psi} \right) = 0$$

$\alpha = -1$

$\alpha \gamma = -1$

$(F\phi)(\phi\phi) = -1$

$F\phi = c_1$
 $\phi\phi = -\frac{1}{c_1}$

$F = \frac{c_1}{f}$
 $\phi = -\frac{1}{c_1\phi}$

$\dot{F} = -\frac{c_1 \dot{f}}{f^2}$
 $\dot{\phi} = \frac{c_1 \dot{f}}{f^2}$

~~$\frac{1}{2} \frac{f}{f} - \frac{1}{4} \frac{f^2}{f} + \frac{1}{4} \frac{f^2}{f^2} = c$~~

$\ddot{f} + \frac{2cc_1}{f} = 0$

$f f'' + 2cc_1 = 0$

$\frac{1}{2} (\dot{f})^2 + 2cc_1 \ln f = 0$

$\dot{f} = \sqrt{-4cc_1 \ln f}$

$\int \frac{df}{\sqrt{-4cc_1 \ln f}} = \int 2\sqrt{-cc_1} dt$

